Stability analysis of low-NOx gas turbine combustors

F. Di Palma, A. Di Benedetto, F. S. Marra

Istituto di Ricerche sulla Combustione - C.N.R., Naples - ITALY

INTRODUCTION

Lean premixed combustion in gas turbine has become widely accepted in recent years as an effective means to control NOx emissions. Lean premixed combustion allows combustion at low temperature thus reducing thermal NOx. Unfortunately, at low equivalence ratio, combustors may become unstable and self-sustained oscillations may arise. These oscillations are undesirable because they cause acoustic noise and reduce the lifetime and regions of operability of the combustors.

Extensive experimental and theoretical work has been done in order to identify the physiochemical mechanisms responsible for the gas turbine combustors instability [1-5], and to predict the critical operating conditions [1, 6-10]. However, these studies do not allow to definitively clarify the causes of the occurrence of the pressure oscillations experimentally observed. Some of them clearly point out the role of coupling between acoustic and heat release from the flame [2-5]. But the experimental results carried out by Richards et al. [1] show that heat losses can have a great influence on the observed oscillations, thus suggesting a thermo-kinetic nature.

The main issue is to understand the role played by each phenomenon and to predict the occurrence of dynamic regimes and its characteristics (amplitudes and frequencies) as function of the operating conditions. For these purposes, simplified models are useful as they allow to isolate the phenomena involved. In these models the combustor is represented as a network of ideal reactors [11, 12], assuming that it can be divided in several zones (mixing and recycling, combustion and post-combustion), each of them being modeled by relatively simple reactors (WSR and PFR). In particular, the combustion zone has been extensively modeled as a well stirred reactor [13, 14], also to study the combustor dynamic behavior [3, 4, 15]. Lieuwen and Zinn [9] recognized the role of the bifurcation theory as a tool able to define the values of the operating parameters at which oscillations arise and to identify the dependence of amplitudes and frequencies on the most important parameters. They made a first attempt to develop the combustor's stability map through experimental investigation.

In a previous work [16], applying the bifurcation theory, we have shown that the lean propane combustion in a well stirred reactor can give rise to stable dynamic regimes. The dynamic of the found oscillations has several similarities with the oscillations observed in Premixed Gas Turbine Combustors. But, a very simple model was adopted, a single WSR, thus adequate to furnishing only the response of a part of the flame to changes of the combustor parameters. In that paper it was shown the relevant role of heat losses from the flame zone in the development of the dynamic regimes concluding that the nature of the oscillations is thermo-kinetic.

A real combustor is much more complex due to spatial non uniformities and coupling with fluid flow and therefore a more adequate model is required. To confirm that heat losses [16] are really important in governing the combustion instabilities in LNGT combustors, in the present paper we developed a more complex model. More specifically, we have included the effect of non homogeneous spatial distributions of composition and temperature and their possible interaction by modeling the combustor as a network of WSR, as suggested in [12]. We have performed the stability analysis of the resulting model in the framework of the

bifurcation theory, investigating the typical operating conditions of a lean premixed combustor.

MATHEMATICAL MODEL DEVELOPMENT

The network of reactors we used was proposed by [12] to model a combustor. The scheme is shown in Fig. 1. In the network each reactor describes a particular zone of the combustor: WSR1, where mixing of reactants occurs, WSR2, where the flame mainly develops, and finally WSR3, second flame zone where combustion is completed. A recycle is established between the exit of WSR2 and the enter of the mixer (WSR1): a part of the hot combustion products is recycled in the mixer to describe the effect of recirculation vortices establishing at the entrance region of the combustion chamber.

Fig. 1 Reactor network describing the combustor model.

The model has been developed under the assumptions of ideal gas behavior and constant specific heats. The model equations are the unsteady balances for total mass, fuel and oxygen mass fractions and energy, written for a constant volume and assuming variable pressure. They read:

Total mass balance

$$\frac{d\rho^{(i)}}{dt}=\frac{1}{V^{(i)}}\left(Q^{Ei}\rho^{Ei}+\sum_{j=1}^{3}R^{j,i}Q^{(j)}\rho^{(j)}-Q^{(i)}\rho^{(i)}\right)$$

Fuel mass fraction balance

$$\frac{dY_f{}^{(i)}}{dt}=\frac{1}{V^{(i)}\rho^{(i)}}\left[Q^{E_i}\rho^{E_i}\left(Y_f{}^{E_i}-Y_f{}^{(i)}\right)+\sum_{j=1}^{3}R^{j,i}Q^{(j)}\rho^{(j)}\left(Y_f{}^{(j)}-Y_f{}^{(i)}\right)+V^{(i)}\nu_f\omega^{(i)}\right]$$

Oxygen mass fraction balance

$$\frac{dY_O{}^{(i)}}{dt}=\frac{1}{V^{(i)}\rho^{(i)}}\left[Q^{E_i}\rho^{E_i}\left(Y_O{}^{E_i}-Y_O{}^{(i)}\right)+\sum_{j=1}^{3}R^{j,i}Q^{(j)}\rho^{(j)}\left(Y_O{}^{(j)}-Y_O{}^{(i)}\right)+V^{(i)}\nu_O\omega^{(i)}\right]$$

Energy balance

$$\frac{dT^{(i)}}{dt}=\frac{1}{V^{(i)}\rho^{(i)}\left(c_p-\frac{R_u}{W}\right)}\cdot$$

$$\cdot\left[Q^{E_i}\rho^{E_i}c_p\left(T^{E_i}-T^{(i)}\right)+\sum_{j=1}^{3}R^{j,i}Q^{(j)}\rho^{(j)}\left(T^{(j)}-T^{(i)}\right)+V^{(i)}\frac{R_u}{W}T^{(i)}\frac{d\rho^{(i)}}{dt}-V^{(i)}\sum_{k=1}^{5}\left(h^0_{f,k}-\frac{R_u}{W}T\right)\nu_k\omega^{(i)}+V^{(i)}q^{(i)}\right]$$

where the superscript *(i)* refers the reactor number *i*, $R^{j,i}$ is the fraction of outlet stream from reactor *j* towards reactor *i* (the only recycle present is $R^{2,1}$), and the superscript E_i is an external inlet stream in reactor *i* (here only $E_1 \neq 0$). The variables ρ, V, Q, T, y_f, y_O are gas density, reactor volume, flow rate, temperature and fraction mass of fuel and oxygen in each reactor respectively. c_p, ν, h^0_f, W, R_u are the constant pressure specific heat, the species stoichiometric coefficient, the standard enthalpy of formation, the mean molecular weight of the mixture and the universal gas constant respectively.

The model reaction is the propane combustion. The reaction rate used in the model is a one step kinetic equation evaluated as [17]:

$$\omega^{(1)} = 0 \qquad \omega^{(i)} = k_0 \left(\rho^{(i)}\right)^{1.75} \exp\left(-\frac{Ta}{T^{(i)}}\right) (Y_f{}^{(i)})^{0.1} (Y_O{}^{(i)})^{1.65} \qquad i = 2,3$$

where k_0 = 2.8e8 kg/m^{3}s and Ta = 15098 1/K. Regarding the heat exchange terms:

$$q^{(i)} = h^{(i)} A^{(i)} \left(T^{(i)} - T_c^{(i)}\right) \qquad i = 1,2,3$$

We have not included the momentum balance equations to express the rate of change of the outlet volume fluxes and we have assumed a constant flow velocity at the outlet section of each reactor assigning the values of the volume fluxes. These should represent the fluxes between each combustor zone as derived by measuring the velocity flow field in the real combustor in a stable steady state. These fluxes should change when varying the operating conditions, but inclusion of the momentum equations in the model greatly increases its complexity, not only by increasing the number of equations but especially changing the mathematical properties. The effect of this simplification will be outlined in the following.

RESULTS

The bifurcation analysis of the model has been performed by means of the software AUTO97, based on the continuation method [14]. Heat transfer coefficient in reactor 2 and recycle fraction were assumed as bifurcation parameters. A fixed value of wall temperature (T_c = 450 K) was set, as well as the volume fluxes in each reactor. Their value ($Q^{(1)}$ = 0.005895, $Q^{(2)}$ = 0.02587, $Q^{(3)}$ = 0.02364, m^3/s) were computed for a typical operating condition of lean combustor, applying the discharge low (thereafter referred as $Q^{(i)}$ computed).

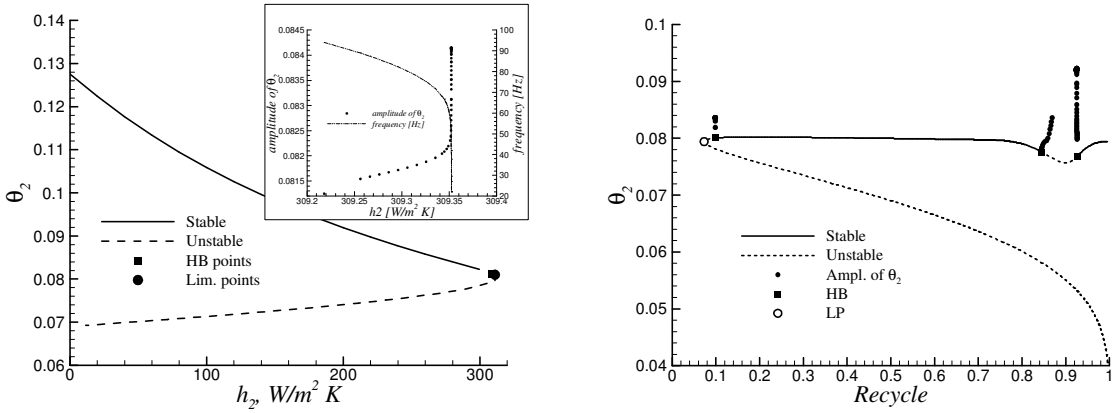

Fig. 2 Bifurcation diagram as function of the heat transfer coefficient in WSR2 (left),and recycle amount (right); Φ = 0.59; Q^{E1} = 0.0055 m^3/s .

The bifurcation diagram as a function of the heat transfer coefficient in the WSR2 ($h^{(2)}$) is firstly illustrated in Figure 2 (left), where the dimensionless temperature of the reactor ($\Theta^{(2)} = T^{(2)}/Ta$) is shown as calculated at a typical operating conditions for a lean combustor (Φ = 0.59; Q^{E1} = 0.0055 m^3/s). It is shown that multiplicity of steady states exists: stable hot steady solutions (solid line), unstable steady states (dashed line) and cold stable steady solutions ($\Theta^{(2)} \approx 0$, not reported for the sake of figure clarity). On increasing the heat transfer coefficient, the temperature in the ignited steady state decreases and just before the blow out condition, a stable Hopf bifurcation point arises (■), leading to stable oscillations. The value of the heat transfer coefficient at which oscillations occur is about 300 W/m^{2}K and it corresponds to reasonable characteristic values of a real combustor [1]. Amplitudes and frequencies of the arising limit cycle are shown in the upper corner of the same figure: the black points represent the maximum value of the dimensionless temperature during

oscillations. On increasing heat losses, amplitudes increase and frequencies decrease, in agreement with the trends experimentally obtained by Richards et al. [1].

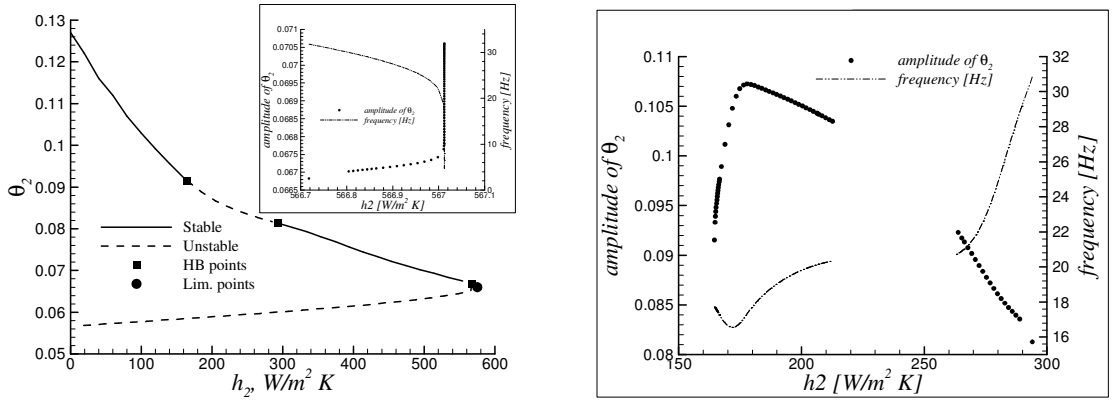

Fig. 3 Bifurcation diagram as function of the heat transfer coefficient in WSR2, $\Phi = 0.59$; $Q^{(i)} = Q^{E1} = 0.0055$ m^3/s; framed plots refer to the oscillating branches.

The effect of changing the recycle is shown in Figure 2 (right), at a fixed values of $h^{(2)} = 323$. At this value, the ignited state cannot be sustained if the recycle is less then a 10% of the burnt product, blow-out still occurring in an oscillating mode: the trend is similar to that found at $R = 0$. Furthermore, the region of existence of oscillations before blow-out shifts to higher values of the heat transfer coefficient and, with respect to this parameter, the range of stable oscillations increases ($309.21 < h^{(2)} < 309.35$ for $R = 0$ and $323.37 < h^{(2)} < 323.59$ for $R = 10\%$). Another region of existence of stable oscillating regimes is found between $R = 82$ and $R = 92\%$, characterized by very high amplitudes (about 20%).

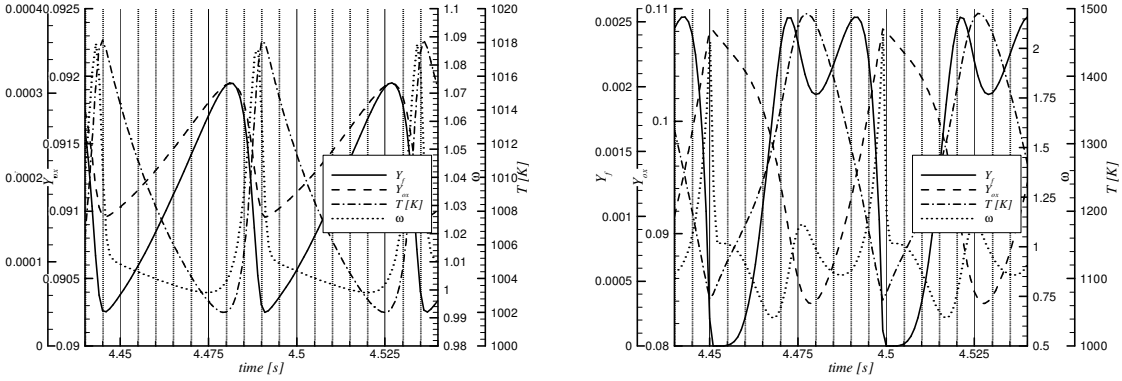

Fig. 4 Temperature, propane and oxygen mass fraction and reaction rate as function of time in WSR2; $h^{(2)} = 567$ W/m^{2}K (left): $h^{(2)} = 240$ W/m^{2}K (right); $\Phi = 0.59$; $Q^{(i)} = Q^{E1} = 0.0055$ m^3/s.

In order to quantify the effect of assuming the volume fluxes equal to the values computed in a specific operating condition, the bifurcation diagram of Figure 2 has been computed under the assumption of constant flow rates in all the system. As shown in figure 3, when $Q^{(i)} = Q^{E1}$, besides the Hopf bifurcation point just before the blow out condition ($h^{(2)} = 574$ W/m^{2}K), other two HB points arise, singling out a new and wider region of stable oscillations ($164 < h^{(2)} < 294$, W/m^{2}K). For the HB point just before extinction, the trend is the same of that found for computed flow rates (amplitude increases and frequency decrease on increasing $h^{(2)}$). A different behavior of oscillations arises for the intermediate range of $h^{(2)}$ (a

not monotone trend). In figure 3 (right) amplitudes and frequencies of the new arising stable limit cycles are shown. To illustrate the different dynamic behavior, simulations were carried out. The mathematical model has been solved adopting an algorithm based on the 4th Runge Kutta scheme. The temperature, the propane and oxygen mass fractions and the reaction rate in WSR2 as function of time for the case of $Q^{(i)} = Q^{E1}$ are reported in Figure 4 for two different values of $h^{(2)}$. As already discussed in [16], near the blow-out conditions, ($h^{(2)} = 567$ W/m^{2}K), the occurrence of the simulated oscillations has to be addressed to the sensitivity of the reaction rate to mixture composition which is enhanced at low values of the equivalence ratio and becomes comparable to that of temperature. In this situation, the positive feedback of the temperature is balanced by the negative feedback of the fuel concentration on the reaction rate. The competition between these two comparable effects causes the oscillations reported in Figure 4 (left). For $h^{(2)}$ in the region of oscillations far from the blow-out point ($h^{(2)} = 240$), a different mechanism is responsible of the dynamic regime. As shown in Fig. 4 (right), the propane and oxygen mass fraction have now opposite phases. Fuel mass fraction goes to zero, indicating complete consumption during a cycle, with oxygen allowed to accumulate and the reactor to cool. In this phase the reaction rate appear sustained by the presence of the oxygen but dramatically decreases when temperature and fuel are lower. It remains low until fuel and temperature accumulate, then start to increase but is firstly limited by the first consumption of fuel and oxygen, then newly sustained by the accumulation of oxygen, leading to the start of a new cycle.

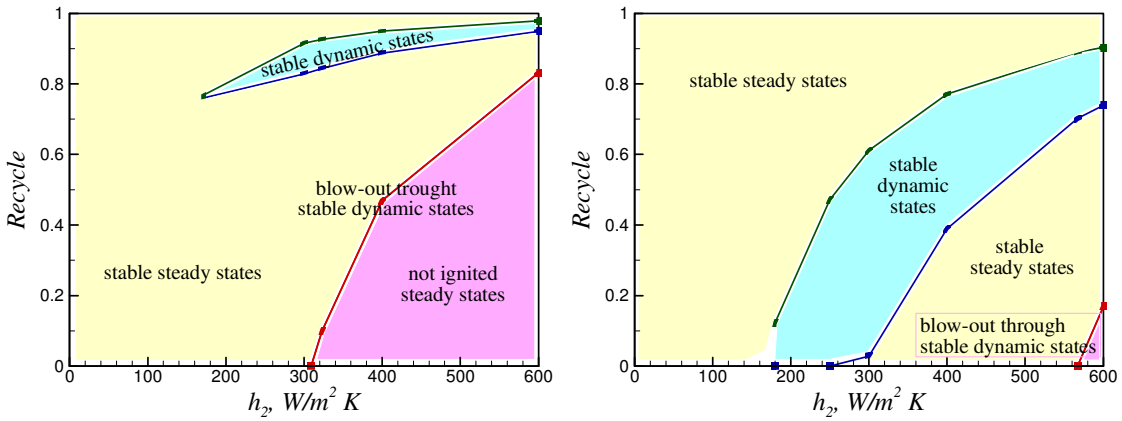

Fig. 5 Stability map as function of the heat transfer coefficient in WSR2 and the recycle. On left, $Q^{(i)}$ computed; on right $Q^{(i)} = Q^{E1}$; $\Phi = 0.59$; $Q^{E1} = 0.0055$ m^3/s.

The above results suggest the relevance of heat exchange, by both the heat losses and the hot gas recycle. To show the interaction between heat losses and recycle we performed a 2 parameters bifurcation analysis. For the case of $Q^{(i)}$ computed (Figure 5 left), for low values of $h^{(2)}$ stable steady states exist for each value of the recycle. At higher values of $h^{(2)}$ ($h^{(2)} = 170$) and high values of the recycles, a region of stable oscillations is present. On increasing $h^{(2)}$, the region of existence of oscillations shifts to higher values of the recycle. Starting from $h^{(2)} = 323$ blow-out may occur and on increasing $h^{(2)}$ the values of R, at which blow out occurs, increase. At $h^{(2)} > 600$ W/m^{2}K, only not ignited solutions exist. In summary, at values of the heat transfer coefficient between 170 and 300 W/m^{2}K, only very high recycle makes the system unstable and oscillations arise, while, increasing $h^{(2)}$, low values of R cause blow-out preceded by stable oscillations. When $Q^{(i)} = Q^{E1}$, (Fig.5, right) the region of ignited states extends, as well as that of stable oscillating solutions that now represent a large stripe dividing the stable steady solution region for low and high values of the heat transfer coefficient.

CONCLUSIONS

By modeling the combustor as a network of WSR, including recycle, we have shown that: the occurrence of an oscillating behavior of a single combustor's zone, found into a narrow region of the space of parameters without any recycle, extends to a wider corresponding region of self-sustained oscillations for the whole combustor; new regions of existence of oscillations are found.
We have also shown that the flow rates strongly affect the dynamic features of the combustor and then it appears advisable to couple the momentum balance equations.

REFERENCES

1. Richards, G.A., Morris, G.J., Shaw, D.W., Keeley, S.A.: and Welter M.J., *Comb. Sci. Tech.*, **94**: 57 (1993).
2. Shih, W.P., Lee, J.G., Santavicca, D.A.: *Proceedings of the Combustion Institute*, **26**:2771 (1996).
3. Lieuwen, T., Neumeier, Y., Zinn, B.T.: *Combust. Sci. Tech*., **135**:193 (1998).
4. Lieuwen T., Zinn B.T.: *Proceedings of the Combustion Institute*, **27**:1809 (1998).
5. Park, S., Annaswamy, A., Ghoniem, A.*: 39th AIAA Aerospace Sciences Meeting & Exhibit*, Reno, NV, January (2001).
6. Broda J.C., Seo S., Santoro R.J., Shirhattikar G., Yang V.: *Proceedings of the Combustion Institute*, **27**:1849 (1998).
7. Richards, G.A., Janus, M.C.: *J. Eng. Gas Turbines Power*, **120**:294 (1998).
8. Johnson C.E., Neumeier Y., Lieuwen T., Zinn B.T.: *Proceedings of the Combustion Institute,* **28**:757 (2000).
9. Lieuwen, T., Zinn, B.T.: *AIAA paper,* 2000-0707 (2000).
10. Steele, R.C., Cowell, L.H., Cannon, S.M., Smith, C.E.: *J. Eng. Gas Turbines Power*, **122**:412 (2000).
11. Lefevbre A.H.: *Gas Turbine Combustor Design Problems*. Mc GRAW-HILL (1978).
12. Swithenbank, J., Poll, I., Vincent, M.W., Wright, D.D.: *Proceedings of the Combustion Institute*, **14**:627 (1972).
13. Zeldovich Y. B., Barenblatt G. I., Librovich V. B., Makhviladze, G. M.: *The Mathematical Theory of Combustion and Explosions*. Plenum Publishing Corp. (1985).
14. Glassman I.: *Combustion: third edition.* Academic Press (1996).
15. Park, S., Annaswamy, A., Ghoniem, A.: *Comb. Flame*, **128**:217 (2002).
16. Di Benedetto, A., Marra, F.S., Russo, G.: *Comb. Sci. Tech.,* **174**:10 (2002).
17. Westbrook, C. K. and Dryer, F. L.: *Combust. Sci. Tech.* **27**: 31 (1981).
18. Doedel E.J., Champneys A.R., Fairgrieve T.F., Kuznestov Y.A., Sanstede B., Wang X.: *AUTO97, Continuation and Bifurcation Software for ODE* (1997).