POD-based Analysis of Cycle-to-cycle Variations in an Optically Accessible Diesel Engine

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1. Introduction

Reducing cyclic variability is the aim of many investigators, seeking thus to modify the appropriate design and operating parameters. A qualitative analysis of cycle-to-cycle variation may aid better understanding. In diesel engines, cyclic variability is due to unsteady in-cylinder flow, and injection variations [1]. Cyclic variability in internal combustion engines is often measured by pressure related parameters, such as maximum cylinder pressure and corresponding crank angle, as well as the coefficient of variation (COV) of the Indicated Mean Effective Pressure (IMEP). Sullivan et al. [2] compare ensemble, cyclic and wavelet-based averaging techniques for velocity measurements.

In-cylinder pressure variability in diesel engines was investigated by means of statistical methods [3] and wavelet transformations [4]. Latest optical systems allow for two- and three-dimensional measurements of in-cylinder variables, resulting in impressive amounts of data necessitating computational data reduction methods. Palacios et al. [5] first applied such methods to optical combustion data, using Proper Orthogonal Decomposition (POD) on video data from a premixed burner, to identify dominant structures and their temporal evolution and separate larger coherent structures and small scale turbulent motion. POD has become a popular tool for analysis of particle image velocimetry (PIV) data from engines and burners. Fogelman et al. [6] applied POD to PIV and computational fluid dynamics (CFD) data from a motored engine; this decomposition may be suitable for construction of a low-dimensional model of the process. Roudnitzky et al. [7] attempted to analyze the velocity field in spark ignition engines, using POD and statistical moments to determine mean, coherent and incoherent parts of the in-cylinder flow.

Although all literature mentioned focuses on the application of POD to velocity measurements, it appears that cycle-to-cycle variations are present in other forms as well, such as combustion light emission. In an earlier paper, POD was used to reconstruct information between consecutive measurements of luminosity fields in an optically accessible spark ignition engine [8], by using a dynamic pressure data weighting procedure, thus overcoming limitations due to the time resolution of the optical setup. In Ref. [9], POD and linear interpolation were applied to flame images from a diesel engine, to reconstruct information between consecutive cycle-resolved measurements.

This paper reports on cycle-resolved, 2D measurements of combustion chamber luminosity in an optically accessible diesel engine. POD is conducted on images acquired from several cycles, reducing the analysis to a small number of scalar coefficients, rather than focusing on the luminosity values of each pixel. Cyclic variability is then analyzed by extracting mean, coherent and incoherent parts of the luminosity field from the POD coefficients, and visualizing their morphology, in a non-truncated representation of the collected data.
2. Experimental

A direct injection Common Rail (CR) four-stroke diesel engine with a single cylinder and a multi-valve production head was used. The research engine features only two valves and utilizes a classic extended piston with an UV grade crown window (34 mm diameter). Thus a full view of the combustion bowl is provided. All the data presented in this paper is referred to use of commercial diesel fuel, without exhaust gas recirculation, an engine speed of 1000 RPM and continuous mode operation. Intake air temperature and pressure were set to 320 K and 1.33 bar, respectively. A typical common rail injection strategy of pre, main and post injections (PMP) [10] in every cycle was used. Pre, main and post injections start at 9°BTDC (before top dead centre), 4°BTDC and 11°ATDC (after top dead centre), and have durations of 400, 625 and 340 µs respectively. Injection pressure was fixed at 600 bar. A typical history of combustion pressure, rate of heat release and drive injector current for the above condition are shown in Fig. 1, where engine operating conditions are also reported. Temporal and spatial evolution of visible flames were investigated by acquiring several images per cycle, with a high speed digital CMOS camera in combination with a 45º UV/visible mirror located inside the elongated piston.

3. Proper Orthogonal Decomposition

Proper Orthogonal Decomposition extracts dominant structures from a given ensemble [11]. When the number of collected samples is smaller than the space discretization, it is more convenient to use the Sirovich approach [12] also known as “method of snapshots”. A given data set \( u_k(x) \), where \( x \) is the space coordinate and \( k \) is the snapshot index, can be conveniently represented as a matrix \( U \equiv u_{jk} \) where \( j=1,\ldots,M \) spans the number of space positions and \( k=1,\ldots,N \) spans the number of snapshots \( u_k \). We can build a set \( \Phi=\{\varphi_1,\varphi_2,\ldots,\varphi_N\} \) of linear combinations of the snapshots:

\[
\varphi_i(x) = \sum_{k=1}^{N} \psi_{ik} u_k(x)
\]

where \( \Psi=\{\psi_1,\psi_2,\ldots,\psi_N\} \) is obtained by solving the eigenvalue problem \( C\Psi = \lambda\Psi \), where \( C \) is the space correlation matrix \( C = (U^T \cdot U)^N \). Then, \( u_k(x) \) can be reconstructed by a linear combination of the N modes:

\[
u_k = \sum_{i=1}^{K} c_{ik} \varphi_i(x)
\]

where \( c_{ik} \) are modal coefficients determined by projection of the data ensemble onto the POD modes. Usually POD modes are ordered according to decreasing magnitude of their corresponding (real, positive) eigenvalues \( \lambda_i \). Cumulative correlation energy is defined as

\[
E_K = \sum_{i=1}^{K} \lambda_i / \sum_{i=1}^{N} \lambda_i
\]

The experimental data set for which POD was performed contains 24 frames, for crank angles from 4° BTDC to 30.5° ATDC, for \( N=37 \) consecutive cycles. Original colour pixels were binned x– and y–wise, to eliminate the colour mask effect and produce a greyscale image of the size of 64x64 pixels. Separate POD basis functions were determined for each data set collected at each specified crank angle, resulting in 37 modes with the method of snapshots.
4. Results and discussion

Figure 2 shows images of main and post injections and the combustion phase at several crank angles. First luminous spots are observed around the injector nozzle at 2.5° BTDC, due to pre injection ignition. The small amount of fuel mixes with a large quantity of air, burns in the cylinder and produces bright spots, due to pockets of fuel that segregate in the chamber. Main injection effectively starts at 2°BTDC and the fuel enters the bowl where the pre injection is already burning. The main injection can be easily investigated because it is lighted by pre combustion flames. This injection occurs in a higher temperature and density environment than the previous one. Jets show a tightened spray angle and fast penetration [13]. In fact, the fuel impinges the bowl surface after 2.5 crank angle degrees. The impact phenomenon is mainly due to the high injection pressure. Jets present a thin liquid core which is not affected by swirl air motion. Main injection combustion occurs at top dead centre (TDC) with an ignition delay of 2° CA (333 µs @ 1000 rpm) from start of injection.

![Figure 2 - Sequence of the combustion images for pre + main + post strategy at 1000 rpm and injection pressure of 600 bar.](image)

<table>
<thead>
<tr>
<th>Single cylinder diesel engine</th>
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<tbody>
<tr>
<td>Engine type</td>
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<tr>
<td>Bore</td>
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<td>Stroke</td>
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<td>Swept volume</td>
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<td>CC volume</td>
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<td>Compression ratio</td>
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<th>Common rail injection system</th>
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<td>Injector type</td>
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<td>Nozzle</td>
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<tr>
<td>Holes number</td>
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<tr>
<td>Cone angle</td>
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<td>Hole diameter</td>
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<td>Rated flow</td>
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![Figure 1 - Optical engine and injection system specifications (left); Fuel injection pattern (dashed), and resulting pressure and heat release rate for a typical cycle (right).](image)
Figure 3 – a) Typical frames at two crank angles, for three combustion cycles; b) Standard deviation of the luminosity field as a function of the crank angle.

The image at 1° BTDC shows the start of luminous combustion on all jets. Subsequently, combustion spreads along the jets. From 2° to 5° ATDC, combustion is present on all jets and in vicinity of the chamber wall. Flames are distributed unevenly along the jet axes, as noted by other authors in simple systems at controlled pressures and temperatures [14, 15]. As fuel along the jet axes is consumed, the combustion zone moves towards the bowl wall, burning the impinged fuel. At combustion end, flames are mainly distributed near the walls. At 14° ATDC, post injection jets are observed. For this injection, autoignition behaviour is different, i.e., it occurs simultaneously along the whole jets, up to their tips, due to the high temperature near the chamber walls. At 15.5° ATDC, new luminous flames are observed, the highest post combustion luminosity being reached at 17° ATDC.

Cycle to cycle variations are observed by analyzing injection and combustion images and in-cylinder pressure signals. In particular, not all jets burn and/or ignite simultaneously, for both main and post injections. During combustion development, flames are unevenly distributed along the jets’ axes, as seen in Figure 4. Application of POD allows decomposing and analysing the considered flow field, by computing some statistical properties of the coefficients. Suppose that each photograph taken at a given crank angle value can be treated as a random process over the cycles. The mean of the luminosity field can be simply computed as 

$$
\bar{u} = \frac{1}{N} \sum_{k=1}^{N} u_k,
$$

where 

$$
u_k = \sum_{i=1}^{N} c_{ik} \varphi_i.
$$

Hence, if we define 

$$
\bar{\varphi}_i = \frac{1}{N} \sum_{k=1}^{N} c_{ik},
$$

then

$$
\bar{u} = \sum_{i=1}^{N} \bar{\varphi}_i.\n$$

The variance of the luminosity field can be computed as 

$$
\sigma_u^2 = \frac{1}{N} \sum_{k=1}^{N} u_k^2 - \bar{u}^2.
$$

Substituting the expressions for 

$$
u_k = \sum_{i=1}^{N} c_{ik} \varphi_i,
$$

and, as such, easily reconstructed for each instance 

$$
k.
$$

In Figure 3b \(\sigma_{\text{norm}}\) shows three distinct peaks, corresponding to the three injection/ignition events. This is a quantitative confirmation of what can be seen in Figures 2 and 3a, i.e., that ignition is a random process whereas developed jet combustion is rather regular and repeatable.

POD allows visualization of fluctuation in the \(k^{\text{th}}\) image, which is 

$$
u_k' = u_k - \bar{u} = \sum_{i=1}^{N} (c_{ik} - \bar{c}_i) \varphi_i = \sum_{i=1}^{N} c_{ik}' \varphi_i
$$

and, as such, easily reconstructed for each instance \(k\).
Figure 4 – Decomposition of the luminosity field for 17° (left) and 14° (right) ATDC, sample No. 1. Fluctuations are represented in absolute value. The w parts contain all modes within the circle in the scatter plots, where some of the distant modes are not show.

It is interesting to further discriminate coherent from incoherent fluctuations and to visualize the relevant morphologies. The idea is that the coherent part includes all fluctuations possessing a somehow structured feature over the cycles (e.g. some luminous spot appearing in most cycles but not in all). The incoherent part should include all fluctuations for which no pattern can be identified over the cycles. According to the Karhunen-Loève theorem [16], since \( u' \) is the realization of a centred stochastic process in some compact set \( \Omega \) satisfying generic regularity properties, it admits a (Proper Orthogonal) decomposition \( u' = \sum c' \phi_i \) where \( c' \) are pairwise uncorrelated random variables and \( \phi_i \) are pairwise orthogonal in \( \Omega \). The POD is in fact the empirical version of the Karhunen-Loève decomposition. Moreover, if \( u' \) is Gaussian, then the random variables \( c' \) are Gaussian and stochastically independent.

We can therefore assume that our fluctuation \( u' \) is composed of a non-Gaussian (=coherent) part \( z \), plus a Gaussian (=incoherent) part \( w \). We may extract the Gaussian part by computing relevant statistical properties of each \( c'_i \), namely skewness \( \gamma_i = \langle (c'_i)^3 \rangle / \sigma_i^3 \) and kurtosis \( \beta_2 = \langle (c'_i)^4 \rangle / \sigma_i^4 \), where \( \sigma_i \) is the standard deviation of \( c'_i \). There will be \( S \) Gaussian coefficients, with \( 0 \leq S \leq N \), that is coefficients with \( (\gamma_i, \beta_2) \) located in a neighbourhood of \( (0,3) \), contributing to part \( w \). The remaining \( N - S \) non-Gaussian coefficients belong to \( z \) :

\[
u' = w + z = \sum_{i=1}^{S} c'_{i,\text{incoh}} \phi_i + \sum_{i=S+1}^{N} c'_{i,\text{coh}} \phi_i
\]

Figure 4 reports one experimental frames \( (u_i) \), the mean field \( \bar{u} \), and its fluctuation \( u'_i \), then the \( z \) and \( w \) parts of \( u'_i \), for 17° and 14° ATDC, respectively. These crank angles were selected as representative of low and high \( \sigma_{\text{norm}} \) respectively, and correspond to those shown in Fig. 3a. At 17° ATDC, the mean part’s spatial distribution resembles the sample experimental frame. The \( w \) part is almost negligible because of the 8 modes contributing to \( w \) contain only 1.42% of the total correlation energy, thus \( u'_i \approx z \) and all fluctuations can be rated coherent. This may be an indication of the fact that jet flame luminosity carries little information about the underlying turbulent flow, to which incoherent behaviour is ascribed in most of literature. At 14° ATDC on the contrary, the mean part does not reproduce the sample frame very well. The fluctuation carries most of the flame shape information. The incoherent
part \( w \) has a distinct “random” spatial distribution, different from that of \( z \). The 8 modes contributing to \( w \) carry 58.37\% of the correlation energy. The flame luminosity field seems to be carrying some information about the underlying turbulent flow. It is important to note that the information obtained by means of the morphologic analysis of the fluctuation is space-resolved. This allows location of high fluctuation intensity regions in the chamber, by cycle-averaging of \( w \) and \( z \), and discrimination between coherent and incoherent parts.

5. Conclusions

Cycle resolved imaging of a multi-injection strategy in a transparent common rail diesel engine was conducted. Cycle-to-cycle variations were observed by analysing injection and combustion images. The computational method was improved significantly by observing that, based on the Karhunen-Loève theorem, statistical analysis may be conducted on the scalar coefficients rather than on the full data set. These coefficients, when adopting the Sirovich approach, are limited in number (37), even in the full (non-truncated) reconstruction. A spatial structure of the coherent part of fluctuation can always be recognized. A Gaussian part of the fluctuation can be clearly identified at those values of the crank angle corresponding to high variability (standard deviation) of the luminosity field over the cycles.

6. References