Application of Proper Orthogonal Decomposition to the Reconstruction of Space and Time Resolved Images of Flames in Internal Combustion Engines

K. Bizon¹, G. Continillo², S. S. Merola³, B. M. Vaglieco³

1. Istituto di Ricerche sulla Combustione - C.N.R., Napoli - ITALY
2. Dipartimento di Ingegneria - Università del Sannio, Benevento - ITALY
3. Istituto Motori - C.N.R., Napoli – ITALY

1. Introduction

Even though relatively high acquisition frequencies are nowadays attained for high resolution images of the combustion chamber, analysis of morphological and geometrical properties of the flame requires high time resolution. Particularly, available techniques provide high space resolution images – but collected over different cycles. Cycle–to–cycle variations and thermal drift are two of the main problems in collecting and using such data to study time evolution of the flame. In fact, whilst frame averaging over different test sessions can be used to compensate for cycle–to–cycle variations at the price of information loss, each test session cannot last too long due to experimental engine overheating and this limits the number of values of the crank angle that can be collected in each test session and, as a consequence, the – even simulated – time sampling rate cannot be so high as it would be desirable.

In earlier works, various interpolation or reconstruction techniques have been proposed and applied to Internal Combustion Engines (ICE) imaging experimental data [1,2]. Generally, reconstruction techniques based on physical concepts have proved to work better as compared to pure mathematical interpolation [3]. In this work, Proper Orthogonal Decomposition (POD) [4] is coupled with interpolation and applied to reconstruct information in between consecutive measurements of flame images taken from an optically accessible Internal Combustion Spark Ignition Engine [5].

2. Experimental setup

Figure 1 shows the experimental apparatus for 2-D digital imaging, along with a schematic diagram and a photograph of the engine used in the present investigation. This is an optically accessible single cylinder ported fuel injection (PFI) spark ignition (SI) engine. The piston is flat and made transparent through a quartz window (Φ=57mm). The optical path includes an inclined (45°) mirror located in the bottom of the engine. The engine has a specially machined cylinder head of a four valve commercial automotive engine with a pent–roof combustion chamber and the spark plug was centrally located. The in–cylinder pressure was measured for each engine cycle with a pressure transducer. It is known that in–cylinder pressure signal is influenced by cycle–to–cycle variation and by heat transfer due to the quartz window [6]. The experimental apparatus is equipped with an Intensified Cooled CCD camera (ICCD) with an array size of 512×512 pixels and 16-bit dynamic range digitization at 100 kHz. The ICCD spectral range spread from UV (180 nm) until visible (700 nm). The ICCD can acquire high space resolution images but not more than one per cycle. This results in CA sequences that do not belong to the same cycle.

3. Proper Orthogonal Decomposition

Suppose we are given a data set, obtained from simulation or experiment, $u_j$, where $j$ denotes
sampling instance and \( u \) is the state vector of our supposed discrete–time dynamical system. In our case, the state vector includes luminosity data at 512\( \times \)512 space locations, plus some “global” properties, such as for example pressure and CA, and the sampling instances are obviously finite. Thus, our sampled data set is a vector-valued function. It is convenient to treat global properties as parameters and restrict our POD analysis to the 512\( \times \)512 values of luminosity. The data set is then conveniently given as a matrix:

\[
U = \begin{bmatrix}
  u_{1,1} & u_{2,1} & \cdots & u_{M,1} \\
  u_{1,2} & u_{2,2} & \cdots & u_{M,2} \\
  \vdots & \vdots & \ddots & \vdots \\
  u_{1,N} & u_{2,N} & \cdots & u_{M,N}
\end{bmatrix}
\]

where \( N \) is the number of positions in the spatial domain and \( M \) is the number of samples. Samples \( u_j \) are often called “snapshots”. It can be shown that a suitable POD basis \( \Phi = \{\phi_1, \phi_2, \ldots, \phi_N\} \) is obtained by solving the eigenvalue problem \( C\Phi = \lambda\Phi \), where \( C \) is the averaged autocorrelation matrix \( C = \langle UU^T \rangle \), and angular brackets denote averaging operation over the number of samples. The elements of \( \Phi \) are the eigenvectors of \( C \), also called POD “modes” or, in our application, “eigenflames”, given the physical interpretation of the state vector (photographs of ICE flames). Thus, the system evolution \( u_t \) along the time coordinate \( t \), in our case related to the crank angle, can be approximated by a linear combination of the first \( K \) eigenflames:

\[
\tilde{u}_t = \sum_{k=1}^{K} a_k(t)\phi_k
\]

where \( K < N \) is the number of modes used for truncation, and the coefficients \( a_k(t) \) are calculated by conducting the orthogonal projection of the data onto the eigenflame set. The luminosity field can then be reconstructed by using different numbers \( K \) of POD eigenfunctions. Luminosity fields for time (crank angle) in between successive snapshots are reconstructed by interpolation of modal coefficients that are assumed to be linear functions of the crank angle.

Figure 1. Schematic diagrams of experimental apparatus for digital imaging. To the right, drawing and photograph of the PFI SI engine.
Figure 2. Engine specifications and Field of view of the combustion chamber.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>399 cm³</td>
</tr>
<tr>
<td>Bore</td>
<td>79.0 mm</td>
</tr>
<tr>
<td>Stroke</td>
<td>81.3 mm</td>
</tr>
<tr>
<td>Connecting rod</td>
<td>143 mm</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>10:1</td>
</tr>
</tbody>
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Figure 3. First six eigenflames

Figure 4. Experimental and reconstructed flames for 16.5 CAD (upper) and 24.5 CAD (lower)

4. Results and discussion

4.1. Constructing the POD basis
In the simplest approach, we apply the POD technique described above to one set of frames collected during one test session for Crank Angle in the range between −5.5 CAD and 26 CAD. Only 32 out of 64 consecutive frames are used in the computations. Figure 3 shows the subset of the six leading POD modes (eigenflames). Figure 4 shows a comparison
between reconstructed flames and the original data for two different values of the crank angle. In both cases 1, 10 and 15 eigenflames are used in the reconstruction. As one can see in the figure, description of luminosity by using only one eigenflame gives surprisingly good quantitative (luminosity) and qualitative (shape) results. It also appears that adding more mode improves description of details.

**4.2. POD/Interpolation**

In order to reconstruct the luminosity field of the flame between consecutive measurements, POD coefficients are interpolated over the CA. Reconstructed luminosity fields are then compared with the available (but not used in the computations) experimental data. Figure 5 shown a sequence of experimental frames (top row) and a sequence of reconstructed frames (bottom row). Particularly, frames reconstructed with POD interpolation in absence of corresponding experimental frames are those in the bottom row at 19, 20, 21, 22 and 23 CAD. These frames show differences from the corresponding experimental frames, but retain morphological likeliness and, moreover, the sequence acquires regularity in time. This indicates that POD/Interpolation can be used to generate pseudo–cycle–resolved sequences.

![Figure 5. Experimental (top row) and reconstructed (bottom row) sequence](image)

**4.3. Weighted POD**

The experimental procedure is able to collect a large number of frames from different cycles and for different values of the CA. Multiple frames can be collected for each value of the CA. In principle, POD reconstruction can be used to provide pseudo–cycle resolved sequences. A basis can be generated from the full experimental data set, and data to be used for projection onto the basis and computation of coefficients as a function of the CA can be also extracted from the full experimental data set. The simplest idea would be to build one average frame for each available value of the CA. This would generate one pseudo–cycle–resolved sequence representing the whole experimental data set. First we build, in common way, POD modes from the ensemble of data that contains 5 sets of successive frames collected over a number of cycles. In order to reconstruct flames of the one “base” cycle, first we need to determine the data set that will somehow describe the time evolution of the flame. This is necessary, since the coefficients of the modes are to be calculated by conducting the projection of that ensemble onto modes. This artificial data set is built by weighting, on the basis of in–cylinder pressure, the data obtained from the experiment. Suppose we want to reconstruct some series of frames for which we have the evolution of pressure $p^*$. The data, for each value of the CA, can be built as follows:

$$U^*_i(x)|_{\text{CA}} = \frac{\sum_{j=1}^{m} w_j U_j(x)|_{\text{CA}}}{\sum_{j=1}^{m} w_j}, \quad \text{where} \quad w_i = 1 - \frac{p_i - p_{\text{CA}}^*}{\max_j |p_j - p_{\text{CA}}^*|}.$$
Figure 6. Available experimental frames as a function of the crank angle and of the instantaneous value of the pressure. Continuous lines represent selected pressure profiles. \( p^* \) denotes the “average” profile.

Figure 7. Weighted POD. Reconstruction of sequence for the average pressure profile

Figure 8. Weighted POD. Reconstruction of sequence for the lowest pressure profile

Figure 9. Weighted POD. Reconstruction of sequence for the highest pressure profile

Here \( p_i \) is the instantaneous value of in–cylinder pressure for frame \( U_i(x)|_{CA} \), and \( p^*_CA \) is the instantaneous value of in–cylinder pressure at the same CA for the cycle we want to
reconstruct. This results in assigning weight equal to one to the frame for which the value of the pressure equals $p^*$ (that can occur at most once for a given series) and lower weights to all other frames according to the distance of corresponding pressure with respect to $p^*$.

We begin with the reconstruction of some set of successive frames that will correspond with the “average” pressure $p^*(t)$ (Figure 6). Here, “average” means that we select that cycle for which the time evolution of pressure $p^*(t)$ has the minimum distance in the sense of Euclidean metrics from the average evolution of pressure computed over the data derived from 2000 cycles. Figure 7 shows the pseudo–cycle resolved sequence obtained by reconstruction with weighted POD in correspondence of the $p^*(t)$ profile. It is observed that the sequence is, again, more realistic than any sampled sequence of experimental frames: this is due to the fact that flame shapes are somehow merged among frames for each CA value. However, for the same reason it appears that fine structures are damped by the weighted averaging procedure. In principle, this procedure can be applied to reconstruct pseudo–cycle resolved sequences for any pressure profile. Figures 8 and 9 show reconstructed sequences for the lowest pressure and the highest pressure respectively. It is seen that sequences are “realistic” in that they provide some continuity to flame propagation appearance. They are also correctly showing less and more intense luminosities respectively, in accordance to the fact that frames are weighted according to the chosen pressure profiles. Figure 9 also shows alternating intensities, and this is in agreement with onset of knocking as detected by pressure time profiles (see fig. 6, high pressure profiles). Again, fine structures are not visible, due to averaging.

5. Conclusions

Proper Orthogonal Decomposition (POD) coupled with interpolation was applied to reconstruct information in between consecutive measurements of flame images, taken during experiments on an optically accessible Internal Combustion Spark Ignition Engine. Reconstructed luminosity fields were compared with the available (but not used in the computations) experimental data. The results were satisfactory in that reconstructed images closely resemble the corresponding missing experimental images.

In a second application, dynamic in–cylinder pressure data collected together with images were used to weight the “snapshots” collected over different cycles. By this means, data from the overall available measurements could be selected as a function of the chosen pressure–time profile and various pseudo–cycle–resolved sequences could be reconstructed.

6. References