Turbulent flames with realistic thermal expansion of burning matter

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INTRODUCTION
The role of the hydrodynamic flame instability (the Darrieus-Landau or DL instability) for premixed turbulent burning in gas turbines and car engines has been widely discussed, but the final answer still remains unknown [1-3]. For a long time it was supposed that the instability influence is minor [1,4]. Now there is mounting evidence that the DL instability is of primary importance for turbulent burning in the flamelet regime, and sometimes this effect may even dominate [5-10]. Of course, this evidence is rather indirect, and one needs a more quantitative investigation to compare the role of external turbulence and the intrinsic flame instability. Unfortunately, direct numerical simulations cannot help in this case, since the characteristic length scale of the flow (10 – 100) cm exceeds the typical flame thickness \(10^{-4} - 10^{-3}\) cm by many orders of magnitude. Renormalization analysis [11,12] may work better when such a large number of different length scales is involved in the problem. The original papers [11,12] developed the renormalization ideas in the artificial context of a flame with zero thermal expansion, when the densities of the fuel mixture \(\rho_f\) and the burnt gas \(\rho_b\) are the same, i.e. the expansion factor \(\Theta = \frac{\rho_f}{\rho_b}\) is equal unity \(\Theta = 1\). However, the assumption of \(\Theta = 1\) is quite far from reality. Instead, laboratory and industrial combustion is accompanied by rather strong expansion of burning gaseous mixture \(\Theta = 5 - 8\). As a matter of fact, it is the density drop that causes the DL instability, and the larger the expansion factor \(\Theta\), the stronger the instability [1-3].

In the present paper we expand the ideas of the works [11,12] to the case of a flame front strongly corrugated both by the external turbulence and the DL instability in the flamelet regime of burning. Assuming self-similar properties of the corrugated flame dynamics we find analytical formulas for the propagation velocity of a strongly turbulent flame. We demonstrate that the DL instability is of principal importance when the integral turbulent length scale is much larger than the cut off wavelength of the instability.

TURBULENCE AND THE DL INSTABILITY WORK SEPARATELY
According to the well-known Clavin-Williams formula [13] obtained in the case of no thermal expansion \(\Theta = 1\) and zero flame thickness, weak turbulence increases velocity of flame propagation \(U_w\) in comparison with the planar flame velocity \(U_f\) as \(\Delta U = U_w - U_f\),

\[
\Delta U / U_f = U_{rms}^2 / U_f^2,
\]  

(1)

where \(U_{rms} = u_{rms}(L_t)\), \(u_{rms}\) is the turbulent rms-velocity depending on the characteristic length scale \(\lambda\), and \(L_t\) is the integral length scale of the turbulent flow. Equation (1) may be also presented with the help of spectral density \(\varepsilon_j(k)\) of the turbulent kinetic energy

\[
\Delta U / U_f = U_f^{-2} \int_{k_i}^{k_f} \varepsilon_j(k) dk \quad \text{with} \quad u_{rms}^2 = \int_k^k \varepsilon_j(k) dk, \quad (2)
\]
where $k_i = 2\pi / L_i$ and $k_v = 2\pi / L_v$ are the wave numbers corresponding to the integral and Kolmogorov (dissipation) length scales, $L_i$ and $L_v$, the former being usually much larger than the latter $L_i >> L_v$, $k_i << k_v$. In the case of the Kolmogorov turbulent spectrum $\varepsilon_i(k) \propto k^{-5/3}$ we find $\varepsilon_i(k) = (2/3)U_{rms}^2 k_i^{2/3} k^{-5/3}$. Equation (1) has been extrapolated to the case of a strongly turbulent flame assuming self-similar properties of the corrugated front [12]. Following [12] we decompose the spectrum of turbulent flame wrinkles into narrow bands, each of them providing similar small increase of the flame front velocity determined by Eqs. (1), (2). Let us designate the velocity of flame propagation corresponding to the wrinkles with the wave numbers above $k$ by $U = U(k)$ and rewrite Eq. (2) in the form

$$dU / U = -\varepsilon_i(k) \, dk / U^2.$$  

Integrating Eq. (3) over the whole turbulent spectrum one obtains the propagation velocity $U_w$ of a strongly corrugated flame with zero thermal expansion $\Theta = 1$, [12]

$$U_w^2 = U_f^2 + 2U_{rms}^2.$$  

Let us consider similar scale-invariant formulas for the case of a flame front corrugated because of the DL instability only, when there is no external turbulence. It has been obtained experimentally [5,6] that a corrugated spherical flame front unstable according to the DL mechanism accelerates with the velocity of flame propagation $U_w$ depending on the characteristic length scale of the hydrodynamic motion $\lambda$ as $U_w = U_f (\lambda / \lambda_c)^D$, where $\lambda_c$ is the cut off wavelength of the DL instability proportional to the flame thickness, and $2 + D$ has been interpreted as fractal dimension of the flame front. According to the experimental measurements [5,6] the fractal excess is approximately $D = 1/3$ for all investigated laboratory flames. Assuming self-similar properties of the fractal cascade we should expect that the “intermediate” velocity of flame propagation $U = U(k)$ produced by the wrinkles with wave numbers in between $k$ and $k_c = 2\pi / \lambda_c$ depends on $k$ as

$$U = U_f (k / k_c)^{-D}.$$  

The last equation may be also presented in the differential form similar to Eq. (3)

$$dU / U = -\varepsilon_{\text{dl}}(k) \, dk$$  

with $\varepsilon_{\text{dl}} = D / k$ for $k < k_c$ and $\varepsilon_{\text{dl}} = 0$ when $k \geq k_c$.

**TURBULENCE AND THE DL INSTABILITY WORK TOGETHER**

In general, when a flame front with realistic thermal expansion propagates in a turbulent flow, then both the DL instability and external turbulence contribute to the velocity increase. In that case the velocity increase depends both on the scaled turbulent intensity $U_{rms} / U_f$ and on the intrinsic flame parameters $\Delta U / U_f = F(U_{rms} / U_f, \Theta; \ldots)$. When turbulence is weak, then the last formula may be reduced to

$$\Delta U / U_f = C_{\text{dl}} + C_i U_{rms}^2 / U_f^2,$$  

where the coefficients $C_{\text{dl}}$ and $C_i$ depend on intrinsic flame parameters. Obviously, the first term in Eq. (7), $C_{\text{dl}}$, describes the velocity increase due to the DL instability only. The second term describes flame response to external turbulence. If flame propagates perpendicular to the axes of turbulent vortices, then the factor $C_i$ is determined by the so-called turbulence-induced solution and equals

$$C_i = (\Theta + 1)^{-1} \frac{16\Theta^3}{4\Theta^2 + (\Theta^2 + 1)^2}.$$  

2.1.2
in the case of a flame of zero thickness [14]. According to Eq. (8) we have \( C_i = 0.2 - 0.4 \) for \( \Theta = 5 - 8 \). Still, preliminary evaluations show that finite flame thickness may increase the coefficient \( C_i \) approximately twice. Besides, the effect of flame propagation along the vortex axis contributes also to the coefficient \( C_i \) [15]. Thus, at present it is reasonable to treat \( C_i \) as a factor close to unity. Taking into account (3), (6), (7) and assuming scale-invariance of flame dynamics, we find the increase of the turbulent flame velocity produced by one narrow band in the turbulent spectrum

\[
dU / U = -\varepsilon_d(k) dk - C_i \varepsilon_i(k) / U^2 \, dk ,
\]

or

\[
\frac{1}{2} \frac{d}{dk} (U^2) = -\varepsilon_d(k)U^2 - C_i \varepsilon_i(k) .
\]

We came to a linear differential equation with coefficients depending on the variable, Eq. (10), which may be solved analytically by a standard method. Below we will consider the solution to Eq. (10) for the most typical case of the DL cut off wave number smaller than the Kolmogorov cut off \( k_i < k_c < k_v \) and for the excess of the fractal flame dimension \( D = 1/3 \). A more general solution to Eq. (19) is presented in [14].

To find the solution to Eq. (19) we have to consider separately two domains of large and small wave numbers (short and long wavelengths), \( k_c < k < k_v \) and \( k_i < k < k_c \), respectively. Solution to Eq. (10) in the domain \( k < k_c \) may be written as

\[
U^2 = C_i \left( \frac{k}{k_c} \right)^{2D} + 2C_i \frac{1}{k_c} \int^k_{k_i} \eta^{2D} \varepsilon_i(\eta) d\eta ,
\]

where the integration constant is determined by the solution in the domain of short wavelength \( C_i = U^2(k_c) \). It has been shown in [10,16] that flame response to external turbulence is quite different for large and small wave numbers. Because of the strong thermal suppression the harmonics of external turbulence with large wave numbers \( k > k_c \) (small wavelength \( \lambda < \lambda_c \)) wrinkle the flame front only slightly and produce almost no increase in the flame velocity \( U(k_c) \approx U_f \) providing the integration constant of Eq. (11) \( C_i \approx U_f^2 \). Of course, such a conclusion holds only for moderate turbulent intensity, which does not modify the inner structure of a flame front. When the inner flame structure is modified, then we go over from the flamelet regime of turbulent burning to the regime of thickened flames, which is beyond the scope of the present paper.

Then in the case of Kolmogorov turbulence \( \varepsilon_i(k) \approx k^{-5/3} \) and \( D = 1/3 \) we find from (11)

\[
U^2 = U_f^2 \left( \frac{k}{k_c} \right)^{2/3} + \frac{4}{3} C_i U_{rms}^2 \left( \frac{k}{k_i} \right)^{2/3} \ln(k_c / k) ,
\]

or

\[
U_w^2 = U_f^2 \left( \frac{L_i}{\lambda_c} \right)^{2/3} + \frac{4}{3} C_i U_{rms}^2 \ln(L_i / \lambda_c) .
\]

When turbulent intensity is zero \( U_{rms} = 0 \), then Eq. (13) goes over to the velocity increase produced by the DL instability only. Comparing the second terms in the velocity increase in Eq. (4) and Eq. (13) (the terms related to the external turbulence) we can see that the turbulent term in Eq. (13) is multiplied now by a large factor \( \ln(L_i / \lambda_c) \), which makes the influence influence of external turbulence much stronger in the presence of the strong DL instability.
The velocity increase Eq. (13) is presented in Fig. 1 for the parameter domain $L_r/\lambda_c = 10^{-3}$ typical for the combustion experiments [4]; the experimental results are shown by the markers. As we can see, the DL instability is of principle importance when the characteristic length scale of the flow (the integral turbulent length scale) exceeds the cut off wavelength of the instability considerably. The obtained analytical results agree rather well with experiments. Still, in order to perform a careful quantitative comparison one has to take into account details of a particular experiment.

**REFERENCES**