MODELLING OF THE CURVATURE TERM OF THE FLAME
SURFACE DENSITY TRANSPORT EQUATION FOR
REYNOLDS AVERAGED NAVIER STOKES SIMULATIONS

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ABSTRACT
Three-dimensional statistically planar simplified chemistry based Direct Numerical Simulations (DNS) of turbulent premixed flames with wide variations of Karlovitz number \( Ka \), heat release parameter \( \tau \) and global Lewis number \( Le \) have been used for the \( a\)-\( priori \) modeling of the curvature term of the generalised Flame Surface Density (FSD) transport equation in the context of Reynolds Averaged Navier Stokes (RANS) simulations. The simulation parameters used in the current study have been chosen in such a manner that both the corrugated flamelets and thin reaction zones regime of premixed turbulent combustion have been considered. The curvature term has been split into the contributions arising due to the reaction and normal diffusion components (i.e. \( T_1 \)) and the term arising due to the tangential diffusion component (i.e. \( T_2 \)). Subsequently, the two sub-terms (i.e. \( T_1^r \) and \( T_2^r \)) of the curvature contributions to the FSD transport have been split into the resolved (i.e. \( T_1^r \) and \( T_2^r \)) and unresolved (i.e. \( T_1^{ur} \) and \( T_2^{ur} \)) contributions. It has been found that \( T_2 \) remains deterministically negative throughout the flame brush. However, the qualitative behaviour of \( T_1 \) varies significantly depending upon the values of \( Ka \), \( \tau \) and \( Le \) considered. Detailed physical explanations have been provided for the observed behaviours of the components of the curvature term. Moreover, it has been observed that the resolved contributions of \( T_1 \) and \( T_2 \) (i.e. \( T_1^r \) and \( T_2^r \)) remains negligible in comparison to the unresolved contributions (i.e. \( T_1^{ur} \) and \( T_2^{ur} \)). Suitable model expressions have been identified for \( T_1^{ur} \) and \( T_2^{ur} \) in the context of RANS simulations, which are shown to perform satisfactorily in all cases considered in the current study, accounting for wide variations in \( Ka \), \( \tau \) and \( Le \).

INTRODUCTION
Flame Surface Density (FSD) based reaction rate closure is one of the most popular methods of the reaction rate closure in turbulent premixed flames in the context of Reynolds Averaged Navier Stokes (RANS) and Large Eddy Simulations (LES) [1-15]. In the context of FSD based formulation the closure of reaction rate translates to the modelling of flame surface area to volume ratio [2]. In the context of RANS simulations the FSD is either evaluated using an algebraic expression [3,9] or a modelled transport equation for FSD is solved alongside other modelled Favre averaged conservation equations [4-12]. The exact
transport equation for the generalised FSD (i.e. $\Sigma_{\text{gen}} = |\nabla c|$) \cite{9, 11-13, 15-17} is given in the following manner \cite{1, 3}:

\[
\frac{\partial \Sigma_{\text{gen}}}{\partial t} + \frac{\partial (\tilde{u}_j \Sigma_{\text{gen}})}{\partial x_j} = -\frac{\partial}{\partial x_j} [(u_i)_s - \tilde{u}_j] \Sigma_{\text{gen}} + \left( (\delta_{ij} - N_i N_j) \frac{\partial u_i}{\partial x_j} \right) \Sigma_{\text{gen}} - \frac{\partial}{\partial x_j} [(S_d N_i) \Sigma_{\text{gen}}] + \left( S_d \frac{\partial N_i}{\partial x_j} \right) \Sigma_{\text{gen}}
\]

where $\tilde{u}_j = \overline{u_i}/\rho$ and $u'_j = u_i - \tilde{u}_j$ are the Favre mean and fluctuating velocity components in the $i^{th}$ direction, $\rho$ is the fluid density, $c$ is the reaction progress variable, $\nabla = -\nabla c/|\nabla c|$ is the local flame normal vector, $S_d = [\tilde{u}_i + \nabla.(\rho D \nabla c)]/\rho |\nabla c|$ is the displacement speed, $D$ is the progress variable diffusivity and $Q_{\text{gen}} = \overline{Q \nabla c} \Sigma_{\text{gen}}$ denotes the surface averaged value of a general quantity $Q$ \cite{6, 9, 11-13, 15-17}, with the overbar suggesting a Reynolds averaging operation. The final term on the right hand side of eq. 1 (i.e. $(S_d \nabla \tilde{N})_{\Sigma_{\text{gen}}}$) arises due to curvature $\kappa = \nabla \tilde{N}/2$ and thus commonly referred to the FSD curvature term. Peters \cite{18} indicated that $\kappa$ dependence of $S_d$ becomes important with increasing Karlovitz number $Ka = (u'/S_L)^{1/2} (l/\delta_{th})^{-1/2}$ where $u'$ is the root-mean-square turbulent velocity fluctuation, $l$ is the integral length scale, $S_L$ is the unstrained planar laminar burning velocity and $\delta_{th} = (T_{ad} - T_0) / \text{Max} \{\tilde{T}\}$ is the thermal laminar flame thickness with $T_{ad}$, $T_0$ and $\tilde{T}$ being the adiabatic flame, unburned gas and instantaneous dimensional temperatures, respectively. The scaling arguments of Peters \cite{18} were subsequently confirmed by DNS data \cite{19}. Moreover, it has been shown that Lewis number (i.e. $Le = \alpha_T/D$ with $\alpha_T$ being the thermal diffusivity) has significant influences on the curvature dependence of displacement speed $S_d$ \cite{19}. Thus, $(S_d \nabla \tilde{N})_{\Sigma_{\text{gen}}}$ is likely to be influenced by the Karlovitz number $Ka$ and Lewis number $Le$ and this needs to be addressed if the model for the FSD curvature term is to be valid for both the corrugated flamelets (where $Ka < 1$) and thin reaction zones (where $100 > Ka > 1$) regimes of combustion for a large range of different Lewis numbers $Le$. In this study the statistical behaviours and the modelling of $(S_d \nabla \tilde{N})_{\Sigma_{\text{gen}}}$ have been analysed using a database of freely propagating statistically planar turbulent premixed flames with a large range of variations in terms of heat release parameter $\tau = (T_{ad} - T_0)/T_0$ (ranging from 2.3 to 4.5), Karlovitz number $Ka$ (ranging from $Ka = 0.54$ to 13.17) and Lewis number $Le$ (ranging from 0.34 to 1.2).

**MATHEMATICAL BACKGROUND AND NUMERICAL IMPLEMENTATION**

To understand the statistical behaviour of the FSD curvature term $(S_d \nabla \tilde{N})_{\Sigma_{\text{gen}}}$ it is useful to split it in the following manner:
\[
(S_d \nabla \bar{N})_r \Sigma_{gen} = T_1 + T_2 = \frac{2(S_r + S_n)\kappa_m}{\tau_i} \Sigma_{gen} - 4(D\kappa_m^2)_{\Sigma_{gen}}
\]  

where \( S_r = \dot{\omega}/\rho |\nabla e|_{\text{c,e}} \) and \( S_n = \bar{N} \nabla (\rho D \bar{N} |\nabla e|)/\rho |\nabla e|_{\text{c,e}} \) are the reaction and normal diffusion components of displacement speed \( S_d = (S_r + S_n) - 2D\kappa_m \) [19-21]. The term \( T_1 \) represents the curvature term due to \((S_r + S_n)\) whereas \( T_2 \) arises due to tangential diffusion component of \( S_d \) (i.e. \( S_r = -2D\kappa_m \)). Equation 2 suggests that the term \( T_2 \) is deterministically negative whereas the term \( T_1 \) depends on the nature of the correlations between \((S_r + S_n)\) and \( \kappa_m \) and between \(|\nabla e|\) and \( \kappa_m \). For the purpose of modeling, the terms \( T_1 \) and \( T_2 \) can further be split in the following manner:

\[
T_1 = \left( \frac{\rho_0 S_L}{\rho} \right) (\partial(N)_r / \partial x_i) \Sigma_{gen} + T_{1ur} \tag{3a}
\]

\[
T_2 = -D(\partial(N)_r / \partial x_i)^2 \Sigma_{gen} + T_{2ur} \tag{3b}
\]

where the terms \( T_{1r} \) and \( T_{2r} \) (\( T_{1ur} \) and \( T_{2ur} \)) are the resolved (unresolved) parts the terms of \( T_1 \) and \( T_2 \) respectively and \( (N)_r = -(\partial e / \partial x_i)/\Sigma_{gen} \) is the \( i \)-th component of flame normal vector.

In a compressible RANS simulation \( \bar{c} \) needs to be extracted from \( \bar{\bar{c}} \) and recently Katragadda et al. [16] and Chakraborty and Cant [17] proposed \( \bar{c} = (1 + \rho g \frac{15}{2} Le^{-0.26} \bar{c} / (1 + \rho g \frac{15}{2} Le^{-0.26} \bar{\bar{c}}) \) which relates \( \bar{c} \) and \( \bar{\bar{c}} \) for both unity and non-unity Lewis number flames under high and low values of Damköhler number \( Da = IS_L / u'\delta_{th} \).

The contribution \( T_1 \) (\( T_2 \)) is likely to weaken (strengthen) with increasing \( Ka \).

**Table 1.** List of parameters for the present DNS database

<table>
<thead>
<tr>
<th>Case</th>
<th>( u' / S_L )</th>
<th>( L / \delta_{th} )</th>
<th>( Re_t )</th>
<th>( Da )</th>
<th>( Ka )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.41</td>
<td>9.64</td>
<td>56.7</td>
<td>6.84</td>
<td>0.54</td>
</tr>
<tr>
<td>B-G</td>
<td>7.5</td>
<td>2.45</td>
<td>47.0</td>
<td>0.33</td>
<td>13.17</td>
</tr>
</tbody>
</table>

\( Le = 1.0 \) (A,B,F) 0.34 (C), 0.6 (D), 0.8 (E) and 1.2 (G).

\( \tau = 2.3 \) (A), 3.0 (B), 4.5 (C-G)

For the present study, a DNS database of statistically planar turbulent premixed flames has been considered. The initial values of normalised root-mean-square turbulent velocity fluctuation \( u' / S_L \), integral length scale normalised by the thermal flame thickness \( I / \delta_{th} \), \( \tau = (T_{ad} - T_0) / T_0 \), \( Le \), Damköhler number \( Da = IS_L / u'\delta_{th} \), Karlovitz number \( Ka = (u' / S_L)^{1/2} (I / \delta_{th})^{-1/2} \) and turbulent Reynolds number \( Re_t = \rho_0 u' \mu_0 \) are given in Table 1. Standard values are chosen for Zel’dovich number \( \beta = T_{aw} (T_{ad} - T_0) / T_{aw}^2 \), Prandtl number \( Pr \) and the ratio of specific heats (i.e. \( \beta = 6.0 \), \( Pr = 0.7 \), \( \gamma = C_p / C_V = 1.4 \)), where \( T_{aw} \) is the
activation temperature. A domain of $118.64\alpha_{T_0}/S_L \times 131.65\alpha_{T_0}/S_L \times 131.65\alpha_{T_0}/S_L$ is taken for case A, which is discretised by a Cartesian grid of $261 \times 128 \times 128$ with uniform grid spacing in each direction [22] where $\alpha_{T_0}$ is the thermal diffusivity in the unburned gas. In case A, inlet and outlet boundaries are specified in the mean direction of flame propagation, whereas transverse boundaries are taken to be periodic. In cases B-G, a domain of size $50.64\alpha_{T_0}/S_L \times 50.64\alpha_{T_0}/S_L \times 50.64\alpha_{T_0}/S_L$ is discretised using a uniform grid of $230 \times 230 \times 230$ [6-8]. The domain boundaries in the direction of mean flame propagation in cases B-G are taken to be partially non-reflecting and the transverse boundaries are assumed to be periodic. In case A, a $6^{th}$ order central-difference scheme has been used for spatial discretisation in the direction of mean flame propagation, which gradually reduces to an one-sided $4^{th}$ order scheme near non-periodic boundaries whereas a spectral method is used for spatial discretisation normal to the mean direction of flame propagation [23]. In cases B-G a $10^{th}$ central difference scheme is used for internal grid points and the order of differentiation gradually reduces to a $2^{nd}$ order one-sided scheme near non-periodic boundaries [11-13, 15-17]. The time advancement for all viscous and diffusive terms in case A is carried out using an implicit solver, whereas the convection terms in case A and all the terms in cases B-G are time advanced with the help of a third order Runge-Kutta method [11-13,15-17,23]. For all cases, the flame is initialised by a steady unstrained planar laminar flame solution and the turbulent fluctuating velocity field is initialised based on an incompressible homogeneous isotropic velocity distribution, which is generated using a standard pseudo-spectral method [24]. The grid resolution is determined by the resolution of the flame structure, and about 10 grid points are kept within $\delta_{th}$ for all cases considered here.

In all cases flame-turbulence interaction takes place under decaying turbulence and simulations have been carried out for $t_{sim} = \text{Max}(t_f, t_c)$, where $t_f = 1/u'$ is the initial eddy turn-over time and $t_c = \delta_{th}/S_L$ is the chemical time scale. The simulation in case A was run for about $4t_f$, whereas cases B-G was run for a time equivalent to $3.34t_f$. The simulation time remains either greater than (case A) or equal to (cases B-G) one chemical time scale. The simulation time remains comparable to several previous studies [9,11, 13-17, 19-22]. The turbulent kinetic energy and its dissipation rate in the unburned gas ahead of the flame were not varying significantly with time when statistics were extracted for all cases. The Reynolds/Favre averaged quantities are assumed to be function of the co-ordinate in the direction of mean flame propagation ($x_1$ direction) and are evaluated by ensemble averaging the relevant quantities in the transverse directions ($x_2 - x_3$ planes).

Figure 1. The contours of reaction progress variable $c$ at the central $x_1 - x_3$ plane for cases (a) A, (b) C, (c) E, (d) F and (e) G.
RESULTS AND DISCUSSION

The contours of $c$ at the central $x_1$-$x_3$ plane for cases A, C, E, F, G are shown in Figs. 1a-e and the cases B and D are not shown for its similarity to cases F and C respectively. A comparison between Figs. 1a and b reveals that $c$ isosurfaces are parallel to each other in case A, whereas, in case F the $c$ isosurfaces representing the preheat zone (i.e. $c < 0.5$) are significantly distorted and are not parallel to each other, though the $c$ contours representing the reaction zone (i.e. $0.7 \leq c \leq 0.9$) are parallel to each other and less distorted in comparison to the $c$ contours in the preheat zone. As combustion takes place in the corrugated flamelets regime in case A, energetic turbulent eddies cannot penetrate into the flame and flame gets wrinkled only by large-scale fluid motion. By contrast, in cases B-G, combustion takes place in the thin reaction zones regime where the energetic turbulent eddies penetrate into the flame, as flame thickness remains greater than the Kolmogorov length scale though the reaction zone retains its quasi-laminar structure. Figures 1b-e demonstrate that the extent of flame wrinkling increases with decreasing $Le$, which leads to increasing burning rate and flame area generation with decreasing $Le$ in accordance with several previous studies [6,17,25-30]. The increasing burning and flame area generation rates with decreasing $Le$ for the cases C-G are presented elsewhere [17,30] and are not repeated here for the sake of brevity.

The variations of the (normalized) curvature terms $T_1, T_2, T_{1r}, T_{2r}, T_{1ur}, T_{2ur}$ and $(T_1 + T_2) = (S_d \nabla \cdot \bar{N})_s \Sigma_{gen}$ with $\bar{c}$ are shown in Figs. 2a-f for cases A,B,C, E-G. In this and subsequent figures case D is not shown explicitly due to its qualitative similarity to case C. Figures 2a-f demonstrate that contributions of $T_{1r}$ and $T_{2r}$ remain negligible in comparison to $T_1$ and $T_2$ respectively and the terms $T_1$ and $T_2$ remain almost equal to $T_{1ur}$ and $T_{2ur}$ respectively. The contributions of $T_2$ and $T_{2ur}$ remain deterministically negative for all the cases. The magnitude of $(\kappa_w^2)_{s} \times \delta_h$ increases with decreasing (increasing) $Le$ ($Ka$) because of the increased extent of flame wrinkling which contributes to the increase of the magnitudes of $T_2$ and $T_{2ur}$. This can be substantiated from the variations of $(\kappa_w^2)_{s} \times \delta_h$ and wrinkling factor $\Xi = \Sigma_{gen} / |\nabla \bar{c}|$ with $\bar{c}$, as shown in Figs. 3a and b.

Figures 2a-f show that for the $Le<<1$ flames (e.g. case C) the contributions of $T_1$ and $T_{1ur}$ remain positive for the major portion of the flame brush before becoming negative towards the burned gas side. Comparing Figs. 2c-f reveals that the extent of positive (negative) contributions of $T_1$ and $T_{1ur}$ decreases (increases) with increasing $Le$. In case A, the contributions of $T_1$ and $T_{1ur}$ remain positive for the major portion of the flame brush. To explain the statistical behaviours of $T_1$ and $T_{1ur}$ it is useful to look into the nature of the variation of $(\kappa_m^2)_{s} \times \delta_h$ with $\bar{c}$ and the correlations of $(S_r + S_n)$ and $|\nabla c|$ with $\kappa_m$. The variations of $(\kappa_m^2)_{s} \times \delta_h$ with $\bar{c}$ for all cases are shown in Fig. 3c which demonstrate that $(\kappa_m^2)_{s}$ remains positive (negative) towards the unburned (burned) gas side of the flame brush.
Figure 2. Variations of $T_1$, $T_{1r}$, $T_{1ur}$, $T_2$, $T_{2r}$, $T_{2ur}$ and $(T_1 + T_2) = (S_d \nabla \bar{N}) \Sigma_{gen}$ with $\tilde{c}$ for cases: (a) A, (b) B, (c) C, (d) E, (e) F and (f) G. The terms are normalised with respect to $S_L / \delta_{th}^2$ in this and subsequent figures.

The correlation coefficients for the $(S_r + S_n) - \kappa_m$ and $|\nabla c| - \kappa_m$ correlations for five different $c$ isosurfaces are shown in Table 2, which show that both $(S_r + S_n)$ and $|\nabla c|$ remain positively (negatively) correlated with curvature for the $Le < 1$ ($Le > 1$) flames which give rise predominantly positive contributions of $T_1$ and $T_{1ur}$ for cases C and D and the transition from positive to negative value for $T_1$ and $T_{1ur}$ takes place towards the unburned gas side for case G. As both $(S_r + S_n)$ and $|\nabla c|$ remain positively correlated with $\kappa_m$ in cases B and F, the curvature terms $T_1$ and $T_{1ur}$ remain positive (negative) towards the unburned (burned) gas side of the flame brush due to the positive (negative) values of $m_k$ towards the unburned (burned) gas side. As both $(S_r + S_n)$ and $|\nabla c|$ remain positively correlated with $\kappa_m$ in case A, the terms $T_1$ and $T_{1ur}$ remain mostly positive for the major portion of the flame brush and assume small negative values close to the burned gas side. The predominant positive
contribution of $T_i$ in cases A, C and D gives rise to predominantly positive values of $(T_i + T_j)$ towards the unburned gas side of the flame brush and this contribution becomes negative towards the burned gas side of flame brush (see Figs. 2a and c). By contrast, the negative values of $T_j$ dominate over the positive values of $T_i$ in cases B, E-G which leads to a negative contribution of $(T_i + T_j)$ throughout the flame brush (see Figs. 2b-f). The explanations for the differences in $\kappa_m$ response of $\left( S_r + S_n \right)$ and $|\nabla c|$ for cases A-G have been discussed elsewhere [19, 28, 29] and will not be repeated here for conciseness and only the modeling of $T_{1ur}$ and $T_{2ur}$ will be addressed in this paper. If $2(\kappa_m)_s - \partial \left( \Sigma_{gen} \right)_s / \partial x$, and $(S_r + S_n)$ are scaled with $\left[ 1 - \left( N_k_s / N_k \right) \right] \Sigma_{gen}$ and $S_L$ respectively, the term $T_{1ur}$ scales as:

$$T_{1ur} = -\beta_1 S_L \left[ 1 - \left( N_k_s / N_k \right) \right] (\bar{c} - c^*) \Sigma_{gen}^2 / \bar{c} (1 - \bar{c})^m$$

(4)

Table 2. Correlation coefficients for $(S_r + S_n) - \kappa_m$ and $|\nabla c| - \kappa_m$ correlations across the flame brush.

| Corr. | $(S_r + S_n) - \kappa_m$ | $|\nabla c| - \kappa_m$ |
|-------|--------------------------|--------------------------|
| $c = 0.1$ | 0.055 (A); -0.433 (B); 0.041 (C); -0.90 (D); -0.308 (E); -0.440 (F); -0.504 (G) | 0.765 (A); -0.115 (B); 0.396 (C); 0.170 (D); -0.022 (E); -0.148 (F); -0.195 (G) |
| $c = 0.3$ | 0.089 (A); -0.496 (B); 0.476 (C); 0.525 (D); 0.217 (E); -0.519 (F); -0.708 (G) | 0.813 (A); -0.090 (B); 0.571 (C); 0.458 (D); 0.199 (E); -0.131 (F); -0.338 (G) |
| $c = 0.5$ | 0.698 (A); -0.425 (B); 0.646 (C); 0.753 (D); 0.743 (E); -0.376 (F); -0.627 (G) | 0.400 (A); -0.077 (B); 0.598 (C); 0.638 (D); 0.428 (E); -0.135 (F); -0.622 (G) |
| $c = 0.7$ | 0.710 (A); 0.093 (B); 0.793 (C); 0.882 (D); 0.710 (E); 0.027 (F); -0.393 (G) | 0.660 (A); -0.170 (B); 0.417 (C); 0.733 (D); 0.538 (E); -0.223 (F); -0.814 (G) |
| $c = 0.9$ | 0.940 (A); 0.216 (B); 0.832 (C); 0.897 (D); 0.670 (E); 0.270 (F); -0.063 (G) | 0.878 (A); -0.331 (B); 0.698 (C); 0.708 (D); 0.439 (E); -0.402 (F); -0.878 (G) |

The parameter $\left[ 1 - \left( N_k_s / N_k \right) \right] \Sigma_{gen}$ in eq. 4 ensures that the term $T_{1ur}$ vanishes when the flow is fully resolved while $(\bar{c} - c^*) / \bar{c} (1 - \bar{c})^m$ ensures that the qualitative behaviour of $T_{1ur}$ is adequately captured and the transition from a positive to a negative value of $T_{1ur}$ takes place at the right location. The performance of eq. 4 for $\beta_1 = 11.0 \left[ Le^{1.1} \cdot (1 + Ka_L)^{1/2.6} \right]$, $c^* = 1.29 \left[ Le^{0.9} (1 + Ka_L)^{1/2.1} \right]$ and $m = 1 + \{ 1 / (1 + Ka_L) \}^{1.5}$ (with $Ka_L = (\bar{\varepsilon} \delta_0)^{1/2} / S_{L}^{3/2}$ being the local Karlovitz number, where $\bar{\varepsilon}$ is the dissipation rate of turbulent kinetic energy) is
compared with $T_{\text{lar}}$ obtained from DNS data in Fig. 4, which demonstrate for the model given by eq. 4 predicts $T_{\text{lar}}$ satisfactorily for all the flames considered here.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Variations of (a) $\langle \kappa^2 \rangle_s \times \delta h^2$, (b) $\Xi = \Sigma_{\text{gen}} / |\nabla \tilde{c}|$ and (c) $\langle \kappa^2 \rangle_s \times \delta h$ for cases: A ( ), B ( ), C ( ), D ( ), E ( ), F ( ) and G ( ).}
\end{figure}

According to $\beta_i = 11.0 / \left[Le^{1.1} \cdot (1 + Ka_L)^{1/2.6} \right]$, the contribution of $T_{\text{lar}}$ strengthens with decreasing $Le$ because of the enhanced burning rate. For high values of $Ka_L$ the combustion situation tends towards the broken reaction zones regime and thus the chemical reaction rate effects progressively weaken with increasing $Ka_L$, which is accounted for by the $Ka_L$ dependence of $\beta_i$. The expression of $c^\ast$ is taken to be such that its value increases with increasing $Le$ and local Karlovitz number $Ka_L$, which accounts for the difference in the curvature dependences of $(S_r + S_s)$ and $|\nabla \tilde{c}|$ in response to the changes in $Ka$, $\tau$ and $Le$ as demonstrated in Fig. 2 and Table 2. The term $4(D\kappa^2_s)_{s} - \bar{D}[\partial (N_s)_{s} / \partial x_i]^2$ can be scaled as $D_0[[(\Sigma_{\text{gen}} / |\nabla \tilde{c}|) - 1]^n S_L / \alpha T_0 ]^2$ with $n$ being a model parameter and $D_0$ being the unburned gas diffusivity (i.e. $4(D\kappa^2_s)_{s} - \bar{D}[\partial (N_s)_{s} / \partial x_i]^2 \sim D_0[[(\Sigma_{\text{gen}} / |\nabla \tilde{c}|) - 1]^n S_L / \alpha T_0 ]^2$), because $\langle \kappa^2_s \rangle_s$ increases with increasing $\Xi = \Sigma_{\text{gen}} / |\nabla \tilde{c}|$ (see Figs. 3a and b). The above scaling is used to propose a model for $T_{\text{lar}}$ in the following manner:
where $\beta_2$ and $n$ are model parameters which are expected to increase with $Le$ because of higher extent of wrinkling in small Lewis number flames. The predictions of the model given by eq. 5 with $n = 0.428 \exp(-1.4Le)$, $\beta_2 = 2.67 \exp(-0.75Le)$ are compared with $T_{2ur}$ obtained from DNS data in Fig. 5, which suggest that this model satisfactorily predicts $T_{2ur}$ for all the flames considered here. According to the models given by eqs. 4 and 5, the contribution of $T_{1ur}$ progressively weakens with increasing $Ka_L$ and for large values of $Ka_L$, the FSD curvature term is principally made up of $T_{2ur}$, which is consistent with the modelling argument of Peters [18].

![Graphs a to f showing variations of $T_{1ur}$ (---) with $\tilde{\zeta}$ along with the prediction of eq. 4 (---) for cases: (a) A, (b) B, (c) C, (d) E, (e) F, (f) G.](image-url)
Figure 5. Variations of $T_{2th}$ (–) with $\bar{c}$ along with the prediction of Eq. 5 (—) for cases: (a) A, (b) B, (c) C, (d) E, (e) F, (f) G.

CONCLUSIONS

The modelling of the FSD curvature term has been addressed by a-priori analysis of a DNS database of freely propagating statistically planar turbulent premixed flames with wide variations of Karlovitz number, heat release parameter and global Lewis number. In order to propose a model for the FSD curvature term for both the corrugated flamelets and the thin reaction zones regimes, the statistical behaviours of the curvature contributions to the FSD transport due to the combined reaction and normal diffusion component of displacement speed and the tangential diffusion component of displacement speed (i.e. $T_1$ and $T_2$) have been examined separately. It has been shown that the curvature term arising due to the tangential diffusion component of displacement speed $T_2$ remains negative throughout the flame brush for all cases considered here. The curvature term due to the combined reaction and normal diffusion component of displacement speed $T_1$ is shown to be significantly dependent on the regime and the global Lewis number of the prevailing combustion process. It has been found that $T_1$ remains mostly positive for the flames representing the corrugated flamelets regime and the thin reaction zones regime flames with the characteristic Lewis number much smaller than unity whereas this term remains positive (negative) towards the unburned (burned) gas side of the flame brush for the thin reaction zones regime flames with global Lewis number close to unity. The difference in the curvature responses of the
combined reaction and normal diffusion components of displacement speed \((S_r + S_n)\) and the magnitude of reaction progress variable gradient \(|\nabla c|\) in different combustion regimes and for different values of global Lewis number are shown to be responsible for the difference in the behaviour of the curvature term \(T_1\). New models have been proposed for unresolved parts of the curvature terms \(T_1\) and \(T_2\) (i.e. \(T_{ur1}\) and \(T_{ur2}\)) in the context of RANS which are shown to capture qualitative and quantitative behaviours of the FSD curvature term for all the cases considered here. However, the present analysis has been carried out using a simplified chemistry based DNS database with moderate value of turbulent Reynolds number \(Re_t\). Thus further model validations will be required based on experimental and three-dimensional detailed chemistry based DNS data at higher values of \(Re_t\) and finally \textit{a-posteriori} assessment of the models needs to be carried based on actual RANS simulations.

**ACKNOWLEDGEMENTS**

Financial assistance of EPSRC UK is gratefully accepted.

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