SEVENTH MEDITERRANEAN COMBUSTION SYMPOSIUM

A STUDY OF ENTROPY GENERATION IN CONFINED CAVITY AT NATURAL AND THERMOSOLUTAL CONVECTIONS: EFFECTS OF MAGNETIC AND RADIATION PARAMETERS

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Abstract

Natural and thermosolutal convections in a confined cavity filled with air is numerically investigated. The cavity is heated and cooled along the active walls whereas the two other walls of the cavity are adiabatic and insulated. Entropy generation due heat and mass transfers, fluid friction and magnetic effect has been determined in transient state laminar flow by solving numerically: the continuity, momentum and energy equations, using a Control Volume Finite Element Method. The structure of the studied flows depends on six dimensionless parameters which are: the thermal Grashof number, the inclination angle, the irreversibility distribution ratio and the aspect ratio of the cavity. In the presence of a magnetic and radiative effects, two others dimensionless parameters are used which are Hartmann number and Radiation parameter. The obtained results show that entropy generation tends towards asymptotic values for lower thermal Grashof number values, whereas it takes an oscillative behavior for higher values of thermal Grashof number. Transient entropy generation increases towards a maximum value, then decreases asymptotically to a constant value that depends on aspect ratio of the enclosure. Entropy generation increases with the increase of thermal Grashof number, irreversibility distribution ratio and aspect ratio of the cavity. The magnetic field parameter has a retarding effect on the flow in the cavity and this lead to a decrease of entropy generation, Temperature and concentration decrease with increasing value of the magnetic field parameter. As Radiation parameter increases, total entropy increases but this behaviour tends to be suppressed for high values of Hartmann number. At local level, irreversibility charts show that entropy generation is mainly localized on bottom corner of the left heated wall and upper corner of the right cooled wall.

Introduction

The analysis of natural convection in enclosures has received increasing attention due to everyday practices and applications extending from the double paned windows in buildings to the cooling of electronic systems. Double-diffusion natural convection also occurs in a wide range of scientific fields such as oceanography, astrophysics, geology, biology, chemical vapor transformation processes and other fields where temperature and concentration differences are combined. Thermodynamic systems submitted to thermal gradients and friction effects are subjected to energy loss, which induces entropy generation in the system. Many studies have been published concerning entropy generation. Bejan [1] investigated entropy generation phenomena by considering a small 2D element of the fluid as an open thermodynamic system submitted to mass and energy fluxes. Poulikakos and Bejan [2] were interested in rectangular flasks in laminar flow; they showed that entropy generation is proportional to work loss in the system. Magherbi et al. [3] numerically studied entropy generation at the onset of natural convection in a square cavity. They showed that entropy generation depends on thermal Rayleigh number. The effect of irreversibility distribution ratio on entropy generation was also analyzed. Saravanan and Kandaswamy [4] analyzed the convection in a low Prandtl number fluid driven by the combined mechanism of buoyancy and surface tension in the presence of a uniform vertical magnetic field. They showed that the heat transfer across the cavity from the hot wall to cold wall becomes poor for a decrease in thermal conductivity in the presence of a vertical magnetic field. Chamkha, Al-Naser and Sathiyamoorthy [5–8] numerically studied the hydromagnetic double-diffusive convection in a rectangular enclosure with opposing temperature and concentration gradients. A magnetic field and heat generation are imposed. Results showed that the magnetic field reduces the heat transfer and fluid circulation within the enclosure. Also, they concluded that the average Nusselt number increases owing to the presence of a heat sink while it decreases when a heat source is present. The influence of an oriented magnetic field on entropy generation in natural convection flow for air and liquid gallium is numerically studied by Eljery et al. [9], they showed that transient entropy generation exhibits oscillatory behavior for air when Grt ≥ 104 at small values of Hartmann number. Asymptotic behavior is obtained for considerable values of Hartmann number. A transient irreversibility always exhibits asymptotic behavior for liquid gallium. They also showed that heat transfer rate is always described by pure conduction mode for liquid gallium, whereas it presents oscillatory behavior for air Grt ≥ 104. Local irreversibility is strongly dependent on magnetic field direction, magnitude of irreversibility lines increases up to 30°, and then gradually decreases. Entropy generation due to heat/mass transfer of turbulent natural convection due to internal heat generation in a cavity is studied by [10]. It was found that the time-volume averaged Bejan number almost equals one and then decreases quickly against Ra increasing. Though the maximum of entropy generation number increases quickly with Ra, the time-averaged total entropy generation number changes in the opposite trend.

The radiative flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in various propulsion devices for satellites and interplanetary spacecrafts, electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas.The unsteady flow past a moving plate in the presence of free convection and radiation were studied by Mansour [11]. The effect of suction/injection on the flow and heat transfer for a continuous moving plate in a micro polar fluid in the presence of rendition was studied by El-Arabawy [12]. Ibrahim et al [13] studied radiative and thermal dispersion effects on non-Darcy natural convection with lateral mass flux for a non-Newtonian fluid from a vertical flat plate in a saturated porous medium. T.Raja Rani [14] and al studied the effects of radiation, magnetic field, variable viscosity and variable thermal conductivity on similarity solutions of mixed convection at a vertical flat plate embedded in a porous medium are studied numerically. When there is no magnetic field, the parameter *C* takes the value unity and for increasing intensity of the magnetic field, the parameter takes values smaller than unity.

1. Problem statement

Let us consider a two-dimensional square cavity submitted to an inclined magnetic field, $\vec{B}$ with an inclination angle, α from the horizontal plane, shown in Figure 1. The two active left and right walls are at different but uniform temperature and concentration (T'_h, C'_h) and (T'_c, C'_c), respectively while the two other passive walls are insulated and adiabatic. Radiatively fluid nonparticipating medium and all surfaces are gray; the radiative heat flux in y direction is negligible in comparison to that in the x direction. This fluid, air is modeled as a Newtonian, Boussinesq incompressible fluid whose properties are described by its kinematic viscosity ν, its thermal and solutal diffusivities, α_T and D, respectively and its thermal and solutal volumetric expansion coefficients β_T and β_S respectively.

$$\rho = \rho_o \left[1 - \beta_T (T - T_o) - \beta_c (C - C_o)\right] \tag{1}$$

$$\beta_T = -\frac{1}{\rho_o}\left(\frac{\partial \rho}{\partial T}\right)_P \tag{2}$$

$$\beta_c = -\frac{1}{\rho_o}\left(\frac{\partial \rho}{\partial C}\right)_P \tag{3}$$

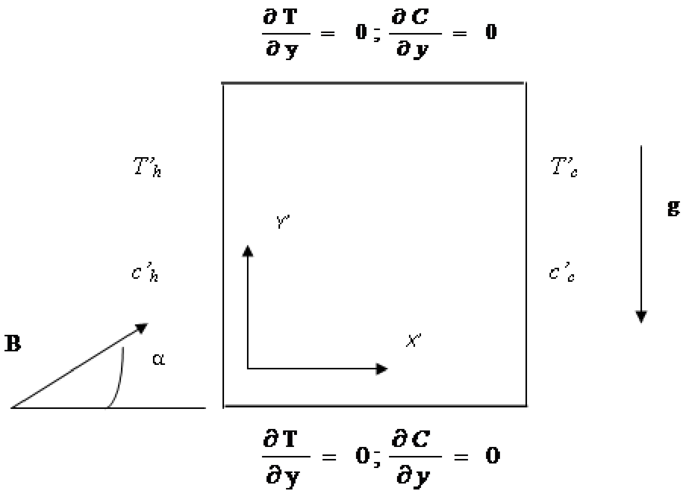


Figure 1: Schematic view of the Physical model

2. Analysis

2.1. Governing equations:

The set of dimensionless governing equations in transient state is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + div\, J_u = -\frac{\partial P}{\partial x} + \Pr . Ha^2 (u \sin\alpha - v \cos\alpha) \sin\alpha \tag{5}$$

$$\frac{\partial v}{\partial t} + div\, J_v = -\frac{\partial P}{\partial y} + Gr(T + N.C) + \Pr . Ha^2 (u \sin\alpha - v \cos\alpha) \cos\alpha \tag{6}$$

$$\frac{\partial T}{\partial t} + div J_T = 0 \tag{7}$$

$$\frac{\partial C}{\partial t} + div J_C = 0 \tag{8}$$

With:

$$J_u = u.V - \Pr \; grad \;\; u$$

$$J_v = v.V - \Pr \; grad \;\; v$$

$$J_T = T.V - (1 + Nr) \, grad \, T$$

$$J_C = C.V - \frac{1}{Le} grad \, C$$

Where the dimensionless variables are defined by:

$$x = \frac{x'}{L}; \; y = \frac{y'}{L}; u = \frac{u'}{U*}; v = \frac{v'}{U*}; t = \frac{t'}{t*}; p = \frac{p'}{p*}; T = \frac{T' - T'0}{\Delta T}; C = \frac{C' - C'0}{\Delta C}$$

$$U* = \frac{\alpha}{L}; \quad t* = \frac{L}{U*}; \quad P* = \rho_0 U^{*2} \;\; ; \quad \Pr = \frac{\nu}{\alpha}; \quad \Delta T = T'_h - T'_c; \; \Delta C = C'_h - C'_c$$

$$Gr_t = \frac{g\beta_T.\Delta T.L^3}{\nu^2}; Gr_C = \frac{g\beta_C.\Delta C.L^3}{\nu^2}, N = \frac{\beta_C.Gr_C}{\beta_T.G_T}; \lambda = \rho\alpha_T Cp$$

$$Ha^2 = \frac{\sigma_e B^{\,2} L^2}{\mu}; \Pr = \frac{\nu}{\alpha_T}; Nr = \frac{4\sigma_0 T_0^3}{3\lambda k^*}; Le = \frac{\alpha_T}{D} \qquad (9)$$

2.2. Boundary and initial conditions:

The boundary conditions appropriate to laminar flow within the differential heated inclined rectangular cavity are:

Hydrodynamic and initial boundary conditions:

at, t = 0 for whole space:

u = v = 0; P=0; C=0.5-x and T = 0.5-x

Thermal and diffusive boundary conditions:

Adiabatic walls:

$$\frac{\partial \varphi}{\partial y} = 0 \text{ On plane y = 0 and y = 1} \qquad (10)$$

Active walls:

$\varphi = 0.5$ On plane x = 0

$$\varphi = -0.5 \qquad \text{On plane} \qquad x = 1 \tag{11}$$

φ: Physical parameter representing temperature or concentration

2.3. Entropy generation

$$\dot{S}_{gen} = \frac{\lambda\left(\nabla \vec{T}\right)^2}{T^2} - \frac{1}{T}\sum_i \vec{J}_{i\,i}\,\nabla\mu_i + \frac{\vec{\tau} : \overline{\overline{grad}}\,\vec{v}}{T} + \frac{\sigma_e\left|\vec{v}\,\Lambda\,\vec{B}\right|^2}{T} \tag{12}$$

The local entropy generation can be putted in a dimensionless form by using the dimensionless variables listed in Equation (9) with considering the temperature and the concentration in the denominator of the entropy generation equation as constants and are equal to bulk temperature(T_0) and bulk concentration(C_0) respectively:

$$\begin{aligned}\mathrm{N}_{s,\ell} = &\left[\left(\frac{\partial \mathrm{T}}{\partial x}\right)^2 + \left(\frac{\partial \mathrm{T}}{\partial y}\right)^2\right] + \lambda_1\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] + \\ &\lambda_2\left[\left(\frac{\partial C}{\partial x}\right)^2 + \left(\frac{\partial C}{\partial y}\right)^2\right] + \lambda_3\left[\left(\frac{\partial \mathrm{T}}{\partial x}\right)\left(\frac{\partial C}{\partial x}\right) + \left(\frac{\partial \mathrm{T}}{\partial y}\right)\left(\frac{\partial C}{\partial y}\right)\right] + \lambda_4 \cdot \left(u \sin\phi - v \cos\phi\right)^2\end{aligned} \tag{13}$$

λ_i are called irreversibility distribution ratios. λ_1 is a ratio of viscous entropy generation by heat transfer entropy generation, while λ_2 and λ_3 are ratios of diffusion entropy generation by heat transfer entropy generation. λ_4 is the ratio of magnetic entropy generation by the same denominator.

The dimensionless total entropy generation is the integral over the system volume of dimensionless local entropy generation.

$$S = \int_V \cdot \mathrm{N}_{S.l} \cdot dV \tag{14}$$

The temperature and concentration gradients are computed along the hot wall of the cavity, and then used to calculate the average Nusselt and Sherwood numbers, respectively, as:

$$\overline{\mathrm{N}u} = \int_0^1 \left(-\frac{\partial T}{\partial y}\right) \cdot dx \tag{15}$$

And

$$\overline{Sh} = \int_0^1 \left(-\frac{\partial C}{\partial y}\right) \cdot dx \tag{22}$$

2.4. Numerical Solution

Energy and Navier-Stokes's equations are solved by determination of the temperature and the velocity scalar fields which depend on the choice of numerical support of resolution. In this study a modified version based on Control Volume Finite Element Method (CVFEM) of Saabas and Baliga [9] is used. To resolve pressure-velocity components, the SIMPLE algorithm (Semi Implicit Method for Pressure Linked Equations) of Patankar [15] is firstly applied, then the SIMPLER algorithm (SIMPLE Revised) and the SIMPLEC approximation of Van Doormal and Raithby in which addition terms of pressure and their relative to velocity are considered, respectively in conjunction with an Alternating Direction Implicit (ADI) scheme for performing the time evolution. The used numerical code written in FORTRAN language was described and validated in details in Abbassi *et al.* [16, 17].

3. Results and Discussions

In this investigation, the Prandtl number was fixed at 0.71. Thermal Rayleigh number, irreversibility distribution ratios and the inclination angle of the magnetic field are in ranges of $10^3 \le Ra \le 10^5$; $10^{-7} \le \lambda_1 \le 10^{-4}$, $10^{-1} \le \lambda_2 \le 0.5$ et $10^{-5} \le \lambda_3 \le 10^{-2}$ and $0° \le \alpha \le 180°$, respectively. λ_2 and λ_3 are equal to 0 in natural convection regime. A grid size of 31×31, 41×41 and 51×51 for respectively $Gr = 10^3$, $Gr = 10^4$ and $Gr = 10^5$ is used. The transient study is carried out with a step time $\tau = 10^{-4}$ for all considered Grashof numbers.

3.1. Case of Natural Convection in absence of Magnetic and Radiation effects

For fixed values of the inclination angle of the enclosure (ϕ = 90°) and the irreversibility coefficient ($\varphi_D = 10^{-2}$), transient entropy generation for $Gr = 10^4$ and 10^5 at different aspect ratio values is illustrated in Figures 2 and 3. As can be seen, entropy generation increases at the beginning of the transient state where the conduction is the dominant mode of heat transfer, reaches a maximum value which is more important as the aspect ratio of the cavity is more important. As time proceeds, entropy generation decreases and tends towards a constant value at the steady state which depends also on the aspect ratio. For low thermal Grashof number, the decrease of entropy generation is asymptotically showing that the system's evolution follows the linear branch of thermodynamics for irreversible process according to Prigogine's theorem. Oscillations of entropy generation are observed for the high values of thermal Grashof number as seen in Figure 4. That is the oscillation behavior obtained before the steady state, corresponds to non linear branch of irreversible processes. In steady state, entropy generation tends towards an asymptotic value which increases with the increase of aspect ratio of the enclosure.

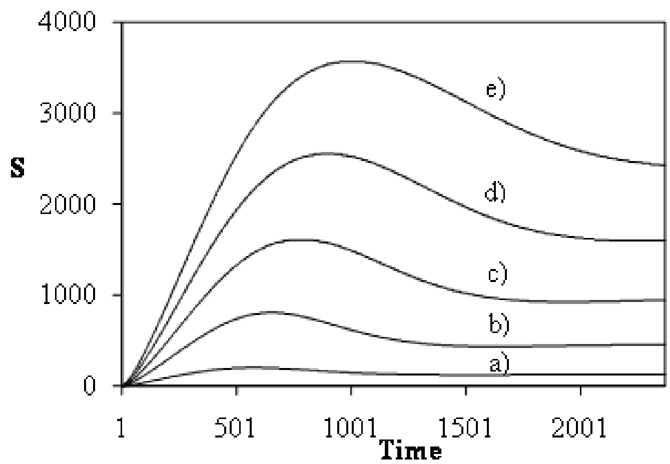


Figure 2: Dimensionless total entropy generation *versus* time for: $Gr = 10^4$; $\varphi_D = 10^{-2}$; ϕ = 90° (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 4; (e) A = 5.

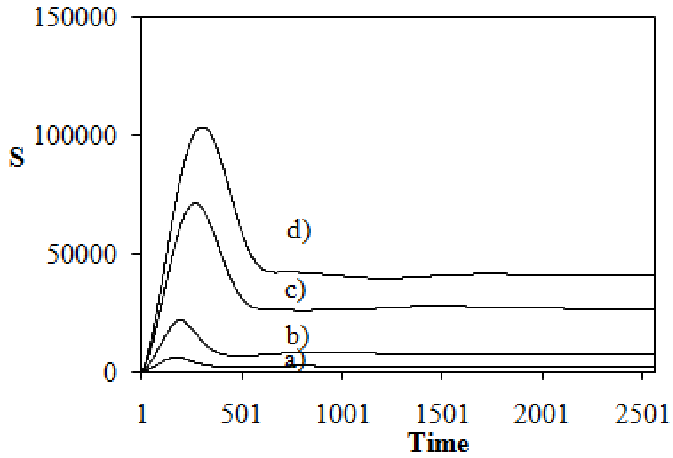


Figure 3: Dimensionless total entropy generation *versus* time for Gr = 10^5; $\varphi_D = 10^{-2}$; $\phi = 90°$; (a) A = 1; (b) A = 2; (c) A = 4; (d) A = 5.

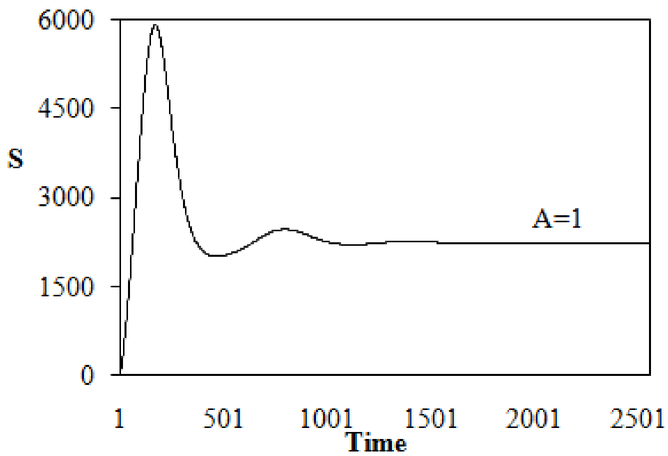


Figure 4: Dimensionless total entropy generation *versus* time for Gr = 10^5; $\varphi_D = 10^{-2}$; $\phi = 90°$; A = 1 (oscillations).

3.2. Case of Thermosolutal Convection with Magnetic effect and absence of Radiation

Figure 5 shows the influence of the magnetic field on the transient entropy generation for the case of natural convection (*i.e.*, N = 0) and for relatively high value of thermal Grashof number ($Gr_t = 10^5$). In absence of the magnetic field (*i.e.*, Ha = 0), entropy generation quickly passes from a minimum value at the very beginning of the transient state towards a maximum value, then exhibits an oscillatory behaviour before reaching a constant value in steady state.

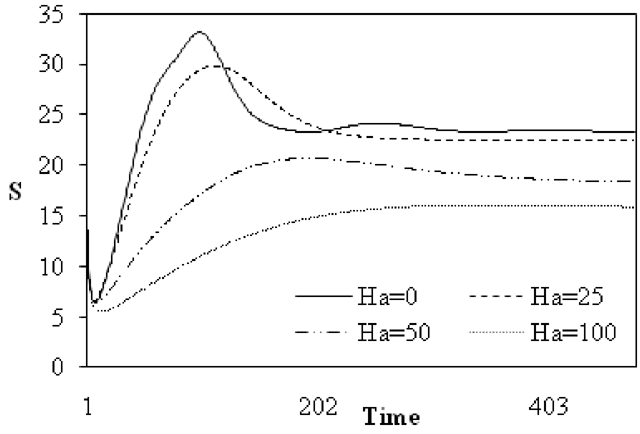
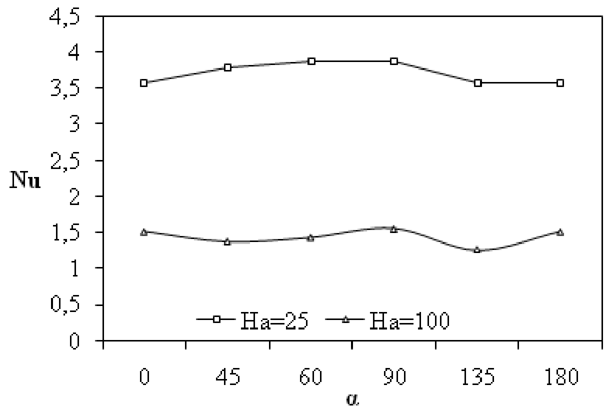


Figure 5: Dimensionless total entropy generation *versus* time for: N = 0; β = 90°; α = 0°; Ra = 10^5.

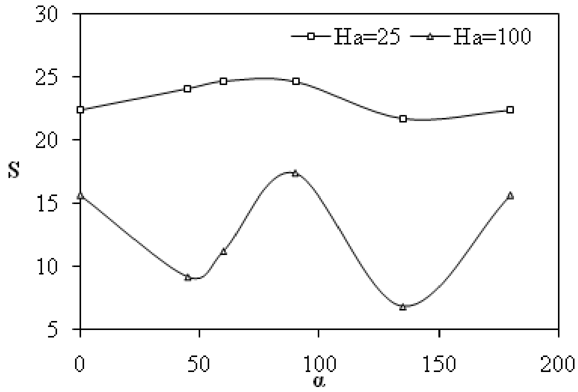
On increasing Hartmann number (*i.e.*, 0 < Ha ≤ 50), a similar situation is also observed but with more flattened maximum entropy generation value and a decreased of both oscillations of transient entropy generation and its magnitude. For higher Hartmann number value (*i.e.*, Ha ≥ 100), entropy generation slowly increases at the beginning of the transient state without oscillations towards a constant value at steady state.

As an important conclusion, the presence of a magnetic field tends to reduce entropy generation where the system passes from an oscillatory behavior describing non linear branch of irreversible process towards an asymptotic behaviour showing the linear branch of thermodynamics for irreversible processes.

Figures 6 illustrates the effect of inclination angle on average Nusselt number and total entropy generation at different values of Hartmann number and thermal Rayleigh number (10^5). Maximum value of Nusselt number is obtained at about α = 90° for which entropy generation is also maximum for both studied Hartmann number values (see Figure 3b). This is due to the increased value of thermal and velocity gradients at this point. Increasing Hartmann number induces the decrease of heat transfer and consequently the dissipated energy expressed by entropy generation. It is important to notice that minimum of entropy generation is obtained for α ≈ 140°.



(a)

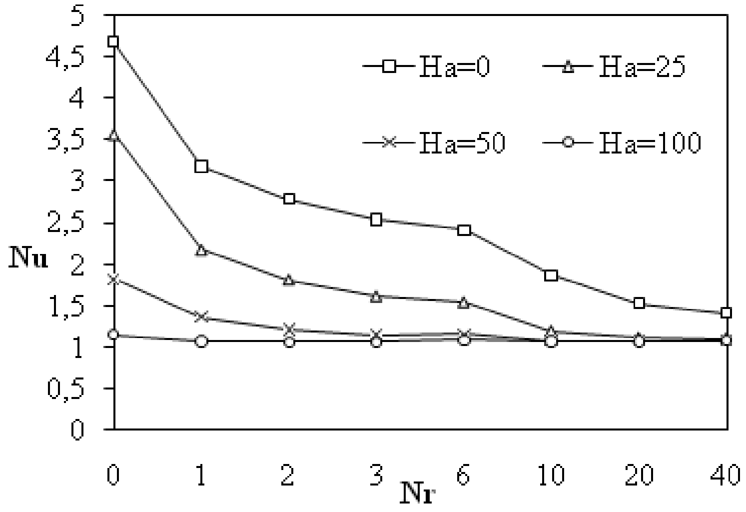


(b)

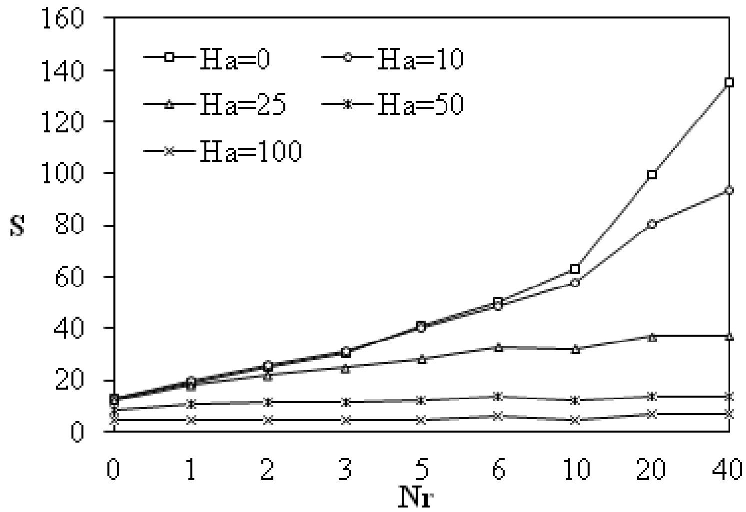
Figure 6. (a) Average Nusselt number and (b) Dimensionless total entropy generation *versus* inclination angle for different Hartmann numbers: Ra = 10^5; β = 90°; N = 0.

3.3. Case of Thermosolutal Convection with Magnetic and Radiation effects

Figure 7 exhibited variation of Nusselt number and total entropy generation rate with Hartmann number and Radiation parameter. Results showed that increasing of these parameters leads to a decreasing of Nusselt number until reaching unity value. Thus radiation effect reduced Nusselt number and homogenized the temperature inside the cavity by reducing the temperature gap between the two insulated walls. It acted to enhance the heat transfer in the cavity. Magnetic effect by Lorentz force had offered a resistance to the flow. Entropy generation rate had increasing with Radiation parameter but it decreased with Hartmann number. As seen in figure 4 and 5, magnetic entropy generation rate and then the viscous one were the main terms those contributing to entropy production which increased and then started constant after a critical radiation parameter(Nr=50). Others terms of entropy generation are insensitive to radiation effect. Total entropy generation was influenced by the buoyancy ratio, a minimum was observed for N=-1 and Nr=0 but this minimum was translated to N=-2 and N=-3 for, respectively Nr=6 and Nr=40. The variation of entropy generation was more pronounced in a cooperate case than in an opposite one where profiles were almost similar.



(a)



(b)

Figure 7. Average Nusselt number (a) and Dimensionless total entropy generation (b) Radiation parameter for different Hartmann numbers: Ra= 10^5; β = 90°; N =0

4. Conclusion

Entropy generation in natural convection through an inclined rectangular cavity is numerically calculated using the Control Volume Finite Element Method (CVFEM). Results show that total entropy generation increases with the aspect ratio of the cavity for high thermal Grashof number, for any fixed irreversibility distribution ratio and with the last parameter at constant Grashof number. The transient state study shows that entropy generation increases at the beginning of this regime, reaches a maximum value, then decreases asymptotically for low Grashof number and with oscillations at high Grashof number towards a constant value at the steady state. At ϕ = 0° and ϕ = 180°, entropy generation takes a constant value depending on aspect ratio of the cavity and corresponding to pure conduction regime by heat transfer. Contributions of thermal, diffusive, friction and magnetic terms on entropy generation are investigated. The magnetic field parameter suppresses the flow in the cavity and this lead to a decrease of entropy generation, temperature and concentration decrease with increasing value of the magnetic field parameter. Entropy generation rate is increasing with Radiation parameter but it decreases with Hartmann number. Total entropy generation was influenced by the buoyancy ratio, a minimum was observed for N=-1, its variation was more pronounced in a cooperate case than in an opposite one. Inclination angle of 80° was the optimum for obtaining maximums values of Nusselt, Sherwood numbers and total entropy generation rate whereas minimums were found for an angle of 135°.

Nomenclature

B	magnetic field (T)
C	dimensionless concentration
C’	concentration ($mol\cdot m^{-3}$)
C'_h	hot side concentration ($mol\cdot m^{-3}$)
C'_c	cold side concentration ($mol\cdot m^{-3}$)
C'_0	bulk concentration ($mol\cdot m^{-3}$)
C_p	specific heat ($J\cdot Kg\cdot K^{-1}$)
D	mass diffusivity ($m^2\cdot s^{-1}$)
g	gravitational acceleration ($m\cdot s^{-2}$)
Gr_t	thermal Grashof number
Gr_c	solutal Grashof number
H (L)	height (length) of the cavity (m)
Ha	Hartmann number
J_k	diffusion Flux (k = u, v, T, C)
k^*	mean absorption coefficient (m^{-1})
Le	Lewis number
N	Buoyancy ratio
Nr	Radiation parameter
Nu	Nusselt number
$N_{s,\ell}$	dimensionless local entropy generation
S	dimensionless total entropy generation
P	pressure ($kg\cdot m^{-1}\cdot s^{-2}$)
Pr	Prandtl number
Ra_T	Rayleigh number
Sc	Schmidt number
Sh	Sherwood number
$\dot{S}_{gen}$	local Volumetric entropy generation ($J\cdot m^{-3}\cdot s^{-1}\cdot K^{-1}$)
T	dimensionless temperature
T’	temperature (K)
t’	time (s)
t	dimensionless Time
T_h’	hot side temperature (K)
T'_c	cold side temperature (K)
T'_o	bulk temperature (K)

u, v dimensionless velocity components
U^* characteristic Velocity ($m \cdot s^{-1}$)
V velocity vector ($m \cdot s^{-1}$)
u', v' velocity components along x', y' respectively ($m \cdot s^{-1}$)
x, y, z dimensionless Coordinates
x', y', z' cartesian coordinates (m)

Greek Symbols

α magnetic field's angle with horizontal direction (°)
α_T thermal diffusivity ($m^2 \cdot s^{-1}$)
β inclination angle of the cavity (°)
β_T thermal expansion coefficient
β_c compositional expansion coefficient
λ_i irreversibility distribution ratios, (I = 1, 2, 3, 4)
μ dynamic viscosity of the fluid ($kg \cdot m^{-1} \cdot s^{-1}$)
ρ fluid density ($kg \cdot m^{-3}$)
σ_0 Stephan constant
σ_e electrical conductivity ($\Omega^{-1} \cdot m^{-1}$)
υ kinematics' viscosity ($m^2 \cdot s^{-1}$)
$\Delta T'$ temperature difference (K)
$\Delta C'$ concentration difference ($mol \cdot m^{-3}$)

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