STABILIZATION OF UNSTABLE COMBUSTION REGIMES USING A FEEDBACK

V.N. Kurdyumov * **and M. Sánchez Sanz** **
vadim.k@ciemat.es
* CIEMAT, Department of Energy
Avda. Complutence 22, ed. 20A, 28040 Madrid, Spain
** ETSI Aeronáuticos, UPM
Pz. de Cardenal Cisneros, 3, 28040, Madrid, Spain

Abstract: In this communication, we consider the possibility of stabilizing intrinsically unstable premixed flames using a feedback. We examine this concept using a premixed flame on a porous-plug burner for values of the parameters that trigger spontaneous flame oscillations. The enforced feedback relates the mass flow rate with the heat flux to the porous-plug burner. A linear stability analysis of the steady states with and without feedback were carried out. We showed that the oscillatory behavior can be suppressed by the feedback and an unstable steady state can be stabilized. The results of the stability analysis were successfully compared with the numerical integration of the time-dependent equations.

Introduction

There were a number of theoretical studies in the past concerned with the onset of instabilities in the form of cells or flame oscillations of the flat premixed flame on a porous-plug burner, see [1]-[3]. Recently, Kurdyumov and Matalon identified in [4] the influence of the mass flow rate, the upstream heat loss to the plug, its thickness and porosity and the Lewis number on the onset of the oscillatory instability by using the constant-density aproximation.

Due to its practical importance, the control of the intrinsic instabilities of premixed flames has always been in mind of researchers and engineers. The papers by Dowling and Morgans [5] and by de Goey et al. [6] review recent theoretical and experimental works in the field of flame stabilization.

Particularly relevant to our research is the analysis carried out by de Goey et al. [6] regarding the control of flames in porous materials. They carried out a stability analysis by introducing a mass flow rate that varied harmonically in time with frequency ω, and a feedback parameter Z defined in terms of the Zeldovich number and of the adiabatic flame temperature. The parametric variation of ω and Z identifies the regions of flame stability.

As an attempt of stabilizing the flame, the study presented below uses the changes in the heat transfer at the surface of the porous material as a feedback parameter to change dynamically the mass flow rate of the mixture of fuel and oxydizer.

Formulation

In the present paper we consider a stream of gas containing a premixture of fuel and oxidizer emerging from a porous plate. The gas stream is uniform with dimensional mass flux $\tilde{m}$. The flame adjust its location relative to that of the plug depending on the fractional mass flux of reactants composing the mixture into the flame and the degree of heat loss to the plate. The thickness of the porous plate is considered to be large compared with the corresponding thermal flame thickness. The thermal conductivity of the plate is assumed sufficiently high to maintain the gas temperature in the plug constant and equal to the upstream temperature T_0. In what follows, x denotes the normal to the porous-plug coordinate while y and z are the transverse coordinates.

The mixture is assumed to be deficient in fuel, so that it is enough to follow its mass fraction, denoted by Y and normalized by its upstream value Y_0. The mass fraction of the oxidizer, which is in abundance, remains nearly constant. The chemical reaction in the gas is modeled by an overall step that converts fuel to products at a mass rate proportional to Y with the Arrhenius temperature dependence, $\tilde{\Omega} = \mathcal{B}\rho^2 Y \exp(-E/R_g T)$, where $\mathcal{B}$, E and R_g is the pre-exponential factor, the activation energy and the universal gas constant. For the sake of simplicity, this paper deals with the well-known diffusive-thermal model, according to which the density of the mixture ρ, the heat capacity c_p, the thermal diffusivity $\mathcal{D}_T$, and the fuel molecular diffusivity $\mathcal{D}$ are all assumed constant.

The enforced feedback consists in the linear relation between the mass flux, $\tilde{m}$, and the heat flux to the porous plug

$$\tilde{m} = \tilde{m}_0 + A\left(\left.\frac{\partial T}{\partial x}\right|_{x=0+} - \tilde{q}_0\right), \tag{1}$$

where $\tilde{m}_0$, $\tilde{q}_0$ and A are prescribed values. If $\tilde{q}_0$ is chosen equal to the temperature gradient at the plug calculated for the steady-state solution with $\tilde{m} = \tilde{m}_0$, then condition (1) does not alter the steady-state.

The laminar flame speed S_L is used below as a unit speed, the thermal thickness of a planar adiabatic flame $\delta_T = \mathcal{D}_T/S_L$ as a unit length and $\mathcal{D}_T/S_L^2$ as a unit time. The dimensionless mass flux becomes $m = \tilde{m}/\rho S_L$ and the dimensionless reaction rate is $\Omega = \mathcal{D}_T\tilde{\Omega}/\rho Y_0 S_L^2$. Using the adiabatic flame temperature $T_e = T_0 + QY_0/c_p$ to define the non-dimensional temperature rise $\theta = (T - T_0)/(T_e - T_0)$, the non-dimensional governing equations in the gaseous phase $(0 < x < \infty)$ become

$$\frac{\partial \theta}{\partial t} + m\frac{\partial \theta}{\partial x} = \Delta\theta + \Omega, \tag{2}$$

$$\frac{\partial Y}{\partial t} + m\frac{\partial Y}{\partial x} = \frac{1}{Le}\Delta Y - \Omega, \tag{3}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. The non-dimensional reaction rate ω takes the form

$$\Omega = \frac{\beta^2}{2Leu_p^2} Y \exp\left\{\frac{\beta(\theta - 1)}{1 + \gamma(\theta - 1)}\right\}. \tag{4}$$

The flame standoff position x_f is defined as the point where the reaction rate reaches its maximum value, $\Omega(x_f) = \Omega_{max}$.

The consideration of the porous-plug adds a difficulty that we avoid by prescribing a fuel mass fraction flux at the surface of the porous plug by means of a Robin-type condition, see [7, 8]. Then, equations (2)-(3) are to be solved for $0 < x < \infty$ with the following boundary conditions

$$x = 0: \quad \theta = 0, \quad m(Y-1) - Le^{-1}\partial Y/\partial x = 0. \tag{5}$$

$$x \to \infty: \quad \partial\theta/\partial x = \partial Y/\partial x = 0. \tag{6}$$

The non-dimensional feedback relation (1) becomes

$$m = m_0 + \alpha\left(\left.\frac{\partial\theta}{\partial x}\right|_{x=0} - q_0\right), \tag{7}$$

where $q_0 = d\theta_0/dx|_{x=0}$ is the extent of (non-dimensional) heat loss to the plug for the steady-state solution ($\partial/\partial t = 0$) calculated with $m = m_0$. Here $\alpha = AQY_0/(\rho c_p S_L)$ is the non-dimensional feedback parameter. The case $\alpha = 0$ represents the porous plug burner without feedback.

The following non-dimensional parameters appear in the above equations: the Zel'dovich number, $\beta = R(T_e - T_0)/R_g T_e^2$, the Levis number, $Le = \mathcal{D}_T/\mathcal{D}$, the heat release parameter, $\gamma = (T_e - T_0)/T_e$ and α is the intensity of the feedback. In what follwos, the Zel'dovich number and the heat release parameter were kept fixed at $\beta = 10$ and $\gamma = 0.7$ considering these values as representative for combustion processes. The factor $u_p = S_L/U_L$ arises in Eq.(4) if the planar flame speed, S_L, is used to define the thermal flame thickness $\delta_T = \mathcal{D}_T/S_L$. Here, $U_L = \sqrt{2\rho\mathcal{B}\mathcal{D}_T\beta^{-2}}\exp(-E/2R_gT_e)$ is the asymptotic value of the velocity of the planar flame calculated in the limit $\beta \gg 1$. The factor u_p is introduced for convenience in order to have the dimensionless planar flame speed equal to one for finite valued of β, see [4] for details.

Steady solutions

For the steady-state solution, denoted hereafter by subindex "0", $\partial/\partial t = 0$ is enforced in Eqs.(2)-(3). The steady-state equations were solved using a Gauss-Siedel method with over-relaxation. The temperature, mass fraction and reaction rate profiles are exemplified in Fig. 1(a), calculated for $Le = 1.2$ and $m_0 = 0.3$. The steady standoff flame position x_f is shown in Fig. 1(b) as a function of the non-dimensional flow rate calculated for $Le = 1.2$. With increasing values of the flow rate, x_f finds its minimum at some intermediate values and then rapidly increases, approaching infinity as $m_0 \to 1$. Anticipating the results of the stability analysis, the solid segment in the figure corresponds to stable states while the dashed segment does to unstable ones. All the results shown here are in agreement with those reported in [4], where the mass transfer inside the porous plug was considered.

Stability analysis

The steady-state solution is perturbed as usual with small harmonic perturbations

$$\begin{aligned} \theta &= \theta_0(x) + \epsilon F(x)\exp(\lambda t + ik_y y + ik_z z), \\ Y &= Y_0(x) + \epsilon G(x)\exp(\lambda t + ik_y y + ik_z z), \end{aligned} \tag{8}$$

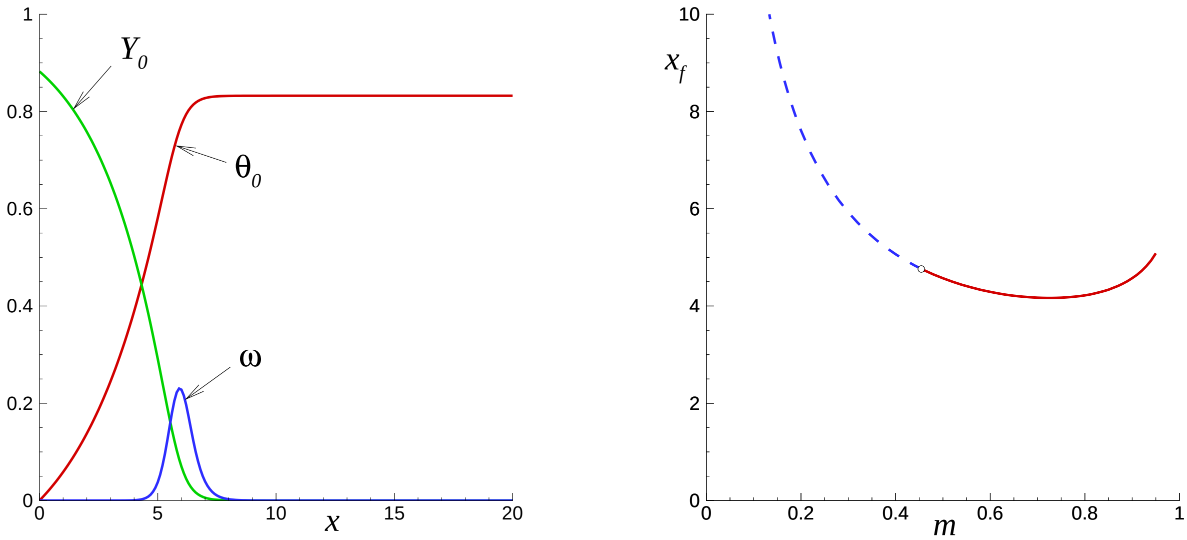

Figure 1: (a) The steady-state distributions of the temperature, mass fraction and reaction rate calculated for $Le = 1.2$, $m_0 = 0.3$, $\beta = 10$ and $\gamma = 0.7$. (b) The dependence of the flame standoff position x_f on m showing the rang of stable (solid segments) unstable (dashed segments) states, for $Le = 1.2$, $\beta = 10$ and $\gamma = 0.7$

where λ is a complex number, whose real part represents the growth rate, k_y and k_z are the wavenumber components and $\epsilon \ll 1$ is the perturbation amplitude. The linearized eigenvalue problem obtained when substituting (8) into the appropriate equations reduces (2)-(3) to

$$\begin{aligned} &\lambda F + m_0 F' + \alpha F'(0)\theta_0' = F'' - k^2 F + AF + BF, \\ &\lambda G + m_0 G' + \alpha F'(0) Y_0' = Le^{-1}(G'' - k^2 G) - AF - BF, \end{aligned} \tag{9}$$

where $k = \sqrt{k_y^2 + k_z^2}$ and

$$A = \frac{\beta^3 Y_0}{2Le u_p^2 [1 + \gamma(\theta_0 - 1)]^2} \exp\left\{\frac{\beta(\theta_0 - 1)}{1 + \gamma(\theta_0 - 1)}\right\}, \quad B = \frac{\beta^2}{2Le u_p^2} \exp\left\{\frac{\beta(\theta_0 - 1)}{1 + \gamma(\theta_0 - 1)}\right\}$$

are both functions of x. The primes denote here and below the differentiation with respect to x.

The boundary conditions at the porous-plug and far downstream take the form

$$\begin{gathered} F(0) = m_0 G(0) - Le^{-1} G'(0) + \alpha F'(0)(Y_0(0) - 1) = 0; \\ F'(\infty) = G'(\infty) = 0. \end{gathered} \tag{10}$$

Stability results

The numerical method described recently in [9] was applied to calculate the eigenvalue with a greatest real part, or the main eigenvalue. This eigenvalue determines completely if

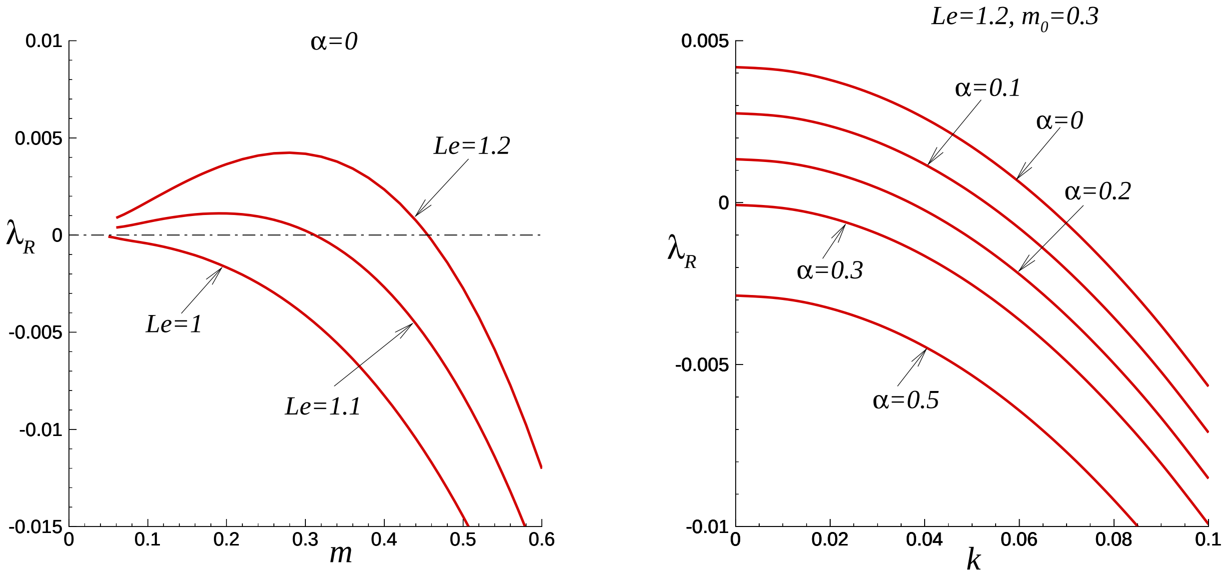

Figure 2: (a) The growth rate λ_R as a function of the mass flux m for several value of Le, computed for $m = 0.3$, $\alpha = 0$ and $k = 0$. (b) The growth rate λ_R as a function of k for several values of the intensity of the feedback, α, computed for $Le = 1.2$ and $m_0 = 0.3$ (unstable steady-state without feedback).

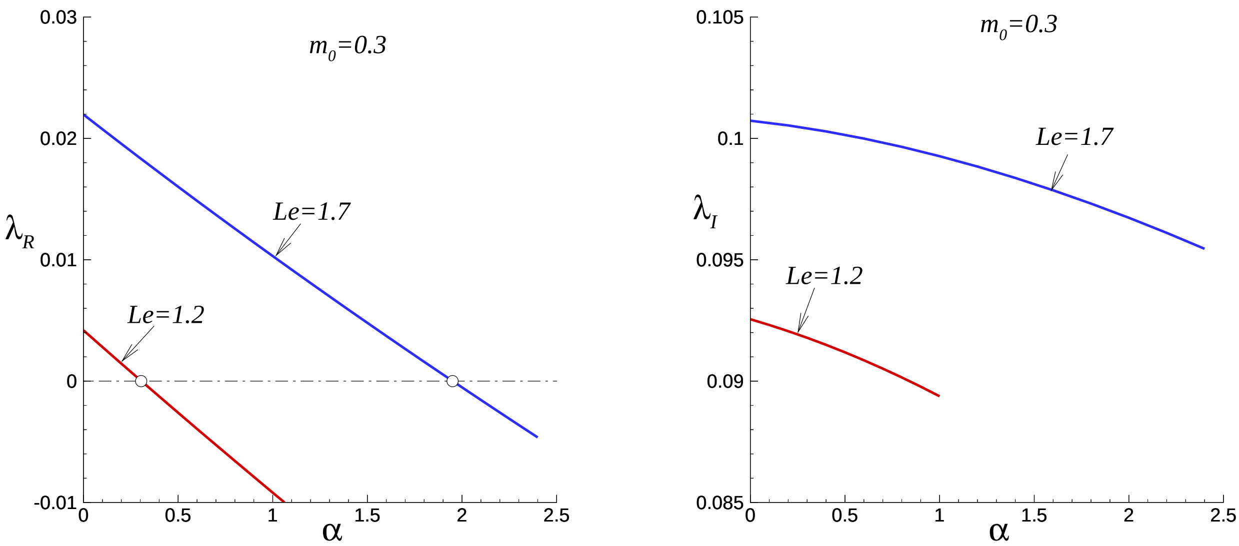

Figure 3: (a) The growth rate λ_R as a function of α computed for two values of Le and $m_0 = 0.3$; the open circles mark the critical values α_c. (b) The frequency of oscillations λ_I as a function of α.

a given steady-state is stable or not. If the real part of this eigenvalue is positive, $\lambda_R > 0$, then the steady-state is unstable, and, conversely, if its real part is nonpositive, $\lambda_R \leq 0$, the steady state is linearly stable.

Consider first the case without feedback, $\alpha = 0$. As it was shown in [4], at criticality the fastest growing wavenumber is $k = 0$, at least for not very high values of the Levis number, implying that the instability is associated with planar pulsations. In Fig. 2(a) the growth rate λ_R is plotted as a function of the flow rate m for different Le. In the equidiffusive case $Le = 1$ the real part of λ is always negative and, therefore, the flame remains stable (for $\beta = 10$, $\gamma = 0.7$). An increase in Le implies positive growth rates λ_R for sufficiently low values of the flow rate.

The dependence of λ_R on the wavenumber k is plotted in Fig. 2(b) as a function of the feedback parameter α. This figure confirms that the fastest growing mode corresponds to $k = 0$ even when the feedback mechanism is considered.

The dependence of λ_R on the feedback parameter α is illustrated in Fig. 3(a) for two values of Le calculated with $k = 0$. As can be seen, when α increases, the growth rate λ_R reduces to dissappear for values of the feedback parameter $\alpha > \alpha_c$, when the flame becomes stable for this given set of parameters. This critical value, shown in Fig. 3(a) with open circles, increases with increasing values of the Lewis number. It is interesting to see that the frequency of oscillations λ_I, plotted in Fig. 3(b), does not change much with the increasing of the intensity of the feedback.

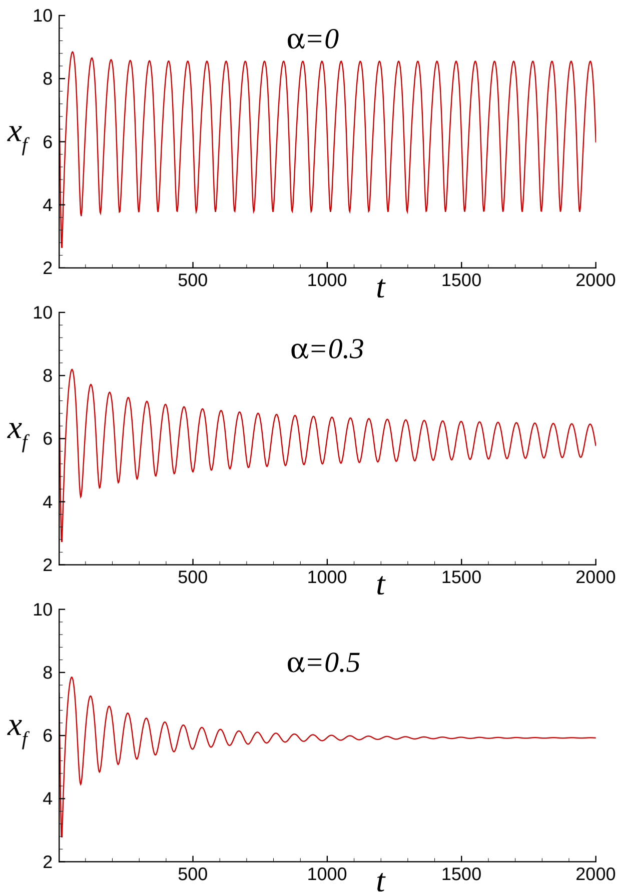

Figure 4: Temporal variations of the flame standoff distance x_f for three increasing values of α. Calculated for $m_0 = 0.3$ and $Le = 1.2$.

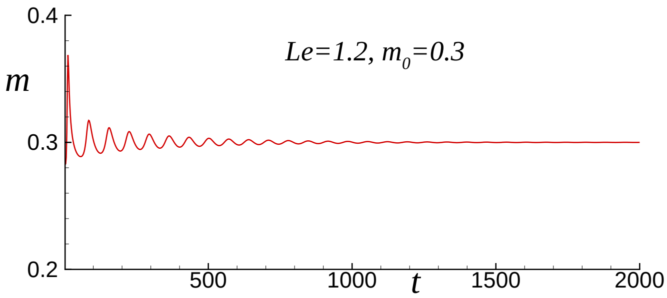

Figure 5: Temporal variations of the flow rate in the case with feedback, calculated for $m_0 = 0.3$, $Le = 1.2$ and $\alpha = 0.5$.

Results of time-dependent simulations

The unsteady nonlinear equations (2)-(3) were integrated numerically subject to the boundary conditions (5)-(6) and the feedback condition (7). In Fig. 4 we show the time-history of x_f for three values of α, all calculated with $Le = 1.2$ and $m_0 = 0.3$. For $\alpha = 0$ the solution evolves to a time-periodic state, with the flame moving back and forth with constant frequency and amplitude. When the feedback parameter is increased (the case $\alpha = 0.3$), the amplitude of oscillations is significantly reduced and the oscillation frequency remains unmodified. Further increase of this parameter $\alpha = 0.5$ leads, after a relatively short transient, to a stable state with the flame located at $x_f \simeq 5.5$. For this particular case, the critical value of the feedback intensity α_c is sightly above 0.3.

Finally, the temporal evolution of the mass flow rate m is shown Fig. 5 for the same set of parameters used above. After the transient, the mass flow rate m achieves the steady state with a a value equal to the steady mass flow rate m_0 initially imposed in condition (7).

Conclusions

This paper represents a first tentative approach to the concept of flame stabilization by using the feedback based on the variations of the heat transfer to the surface of the porous plug to control the mass flow rate.

The stability analysis of the problem has shown a growth rate λ_R greatly affected by the feedback parameter α, reducing the amplitude of the oscillation induced by the thermal-diffusive instabilities. Eventually, for values of $\alpha > \alpha_c$, the real part of the growth rate becomes negative at all values of the wavelength, making the flame unconditionally stable for given values of m_0 and Le. The time-dependent calculation showed the mass flow rate m that oscillates during the stabilization process to achieve a final value equal to that of the steady case m_0 after a some time.

Even though more research is needed to understand completely the implications the feedback parameter α on the stability of the flame, this first initial attempt introduces a technique worth being further considered.

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