

# MATHEMATICAL MODEL OF CROWN FOREST FIRE INITIATION IN THREE DIMENSIONAL SETTING

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## Abstract

Mathematical model of crown forest fire initiation was based on an analysis of experimental data and using concepts and methods from reactive media mechanics. The research was based on numerical solution of three dimensional Reynolds equations. The boundary-value problem is solved numerically using the method of splitting according to physical processes. A discrete analogue for the system of equations was obtained by means of the control volume method. The developed numerical model of crown forest fire initiation would make it possible to obtain a detailed picture of the variation in the velocity, temperature and chemical species concentration fields with time.

## Introduction

As noted in [1,2] the most dangerous type of forest fires are crown forest fires. Therefore, this type of forest fires is important because information on the mechanisms of emergence and development of crown forest fires can be used to develop new ways to prevent and combat them. Discussed in [3,4], describing a two-dimensional forest fires in axially symmetry (no external field wind) and plane approximation (infinite length front surface forest fire and finite width). These approaches have significant limitations. In reality a surface forest fire have finite dimensions, and because of the heterogeneity distribution of forest combustible materials, the front of a forest fire is not a direct line of infinite length. All of these and other restrictions may be lifted if be going down a forest fire in spatial three-dimensional setting. Typically, fire in the forests is in the bottom tier in ground cover (fallen foliage, mosses, lichens, grass, etc.), and then fire covered canopy forest, i.e. crown fire is initiation as a result transition surface fire into crown [1].

## Physical and mathematical model

It is supposed that the centre of a local fire has the finite sizes and over wood bed curtains speed of a wind is set. The beginning of system of co-ordinates  $x_1, x_2, x_3=0$  is connected with the centre of a local fire. Axis  $0.x_3$  facing upwards, and axis  $0x_1$  and  $0x_2$  - parallel to the surface of the Earth ( $x_1$  coincides with the direction of the wind): a scheme for the process provided on (Figure 1).

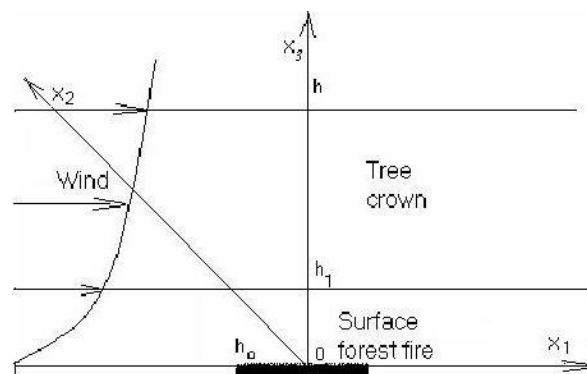


Figure. 1.

Assumes that: 1) for a developed turbulent nature and molecular transfer neglect, 2) density of the gaseous phase does not depend on the pressure of the infinitesimal velocity compared to the speed of sound, 3) environment is in locally-thermodynamic balance, 4) speed of a wind over wood bed curtains in not indignant conditions is known, 5) medium is binary and consists of particles of the condensed phase, and also a gas phase - components of oxygen, gaseous combustible and inert components. Problem formulated above reduces to the solution of systems of equations (1)-(8).

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = \dot{m}, \quad j=1,2,3, \quad i=1,2,3; \quad (1)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (-\rho \overline{v_i v_j}) - \rho s c_d v_i |\vec{v}| - \rho g_i - \dot{m} v_i; \quad (2)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j} (-\rho c_p v_j \overline{T'}) + q_5 R_5 - \alpha_v (T - T_s) + k_g (cU_R - 4\sigma T^4); \quad (3)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j} (-\rho v_j c'_\alpha) + R_{5\alpha} - \dot{m} c_\alpha, \quad \alpha = \overline{1,5}; \quad (4)$$

$$\frac{\partial}{\partial x_j} \left( \frac{c}{3k} \frac{\partial U_R}{\partial x_j} \right) - kcU_R + 4k_s \sigma T_s^4 + 4k_g \sigma T^4 = 0, \quad k = k_g + k_s; \quad (5)$$

$$\sum_{i=1}^4 \rho_i c_{pi} \varphi_i \frac{\partial T_s}{\partial t} = q_3 R_3 - q_2 R_2 + k_s (cU_R - 4\sigma T_s^4) + \alpha_v (T - T_s); \quad (6)$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_1 - \frac{M_c}{M_1} R_3, \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \quad (7)$$

$$\sum_{\alpha=1}^5 c_\alpha = 1, \quad p_e = \rho RT \sum_{\alpha=1}^5 \frac{c_\alpha}{M_\alpha}, \quad \vec{v} = (v_1, v_2, v_3), \quad \vec{g} = (0, 0, g),$$

$$\dot{m} = (1 - \alpha_c) R_1 + R_2 + \frac{M_c}{M_1} R_3 + R_{54} + R_{55},$$

$$R_{51} = -R_3 - \frac{M_1}{2M_2} R_5, \quad R_{52} = v_g (1 - \alpha_c) R_1 - R_5, \quad R_{53} = 0, \quad R_{54} = \alpha_4 R_1, \quad R_{55} = \frac{\alpha_5 v_3}{v_3 + v_{3*}} R_3. \quad (8)$$

Here the symbol  $\frac{d}{dt}$  marked complete derivative:  $\frac{d}{dt} = \frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial x_1} + v_2 \frac{\partial}{\partial x_2} + v_3 \frac{\partial}{\partial x_3}$ . To

determine the rates of reaction of pyrolysis, evaporation, combustion of coke, and volatile products of pyrolysis used formula:

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right), \quad R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right), \quad R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1}\right)^{0.25} \frac{c_2 M}{M_2} T^{-2.25} \exp\left(-\frac{E_5}{RT}\right).$$

where  $R_1 - R_5$ ,  $R_{5\alpha}$ -mass rates pyrolysis forest combustible materials, evaporation, combustion of condensed and volatile products of pyrolysis, soot and ash and  $\alpha$  - components gas and dispersion phase;  $c_{pi}$ ,  $\rho_i$ ,  $\varphi_i$  -specific heat, density and volume fraction of  $i$  - phase (1 - dry organic substance, 2 - water in liquid-water-drip condition, 3-condensed products of pyrolysis, 4- mineral part, 5 - gas phase);  $T, T_s$ -temperature gas and condensed phases,  $c_\alpha$ -

mass concentrations ( $\alpha = 1$ -oxygen, 2-  $CO$ , 3-soot, 4 - ashes, 5-inert components of air);  $p$  - pressure;  $U_R$  -radiation energy density;  $\sigma$  - Stefan-Boltzmann constant;  $k$  -coefficient attenuation of radiation;  $k_g, k_s$  absorption coefficients for gas and condensed phases;  $\alpha_V$  - exchange ratio, phase  $q_i, E_i, k_i$  -thermal effects, activation energy and constants for pyrolysis, evaporation, combustion of coke and volatile products of pyrolysis reactions;  $s_\sigma$ -specific surface element forest combustible materials;  $M_\alpha, M_c, M$  -molecular weight of individual components in the gas phase, carbon and air mixture;  $SC_b$  -specific surface phytomass forest canopy and an empirical factor;  $c$  -speed of light;  $v_i$  -projection velocity on an axis  $x_i$ ;  $\alpha_c, \nu$  -coke number and fraction of combustible gases in the mass of volatile products of pyrolysis;  $\dot{m}$  - mass rate of gas and dispersion phase formation;  $w^*$  -a characteristic of the hearth bottom blowing speed forest fire;  $\alpha_4, \alpha_6$  -empirical constants;  $g$  -acceleration of gravity. System of equations (1)-(7) describes the flow in the area of forest, which includes the space between underlying surface and the bottom edge of the forest canopy, canopy, and the space above the forest canopy. Thermodynamic and thermophysical and structural characteristics of forest combustible materials matches (FCM) of pine forest and can be found in [1-3]. Equations (2)-(4) contain the members of turbulent convection and needed closure. Turbulent stress tensor components and turbulent heat fluxes and mass are recorded via the gradients middle according to [5]. Coefficient of turbulent dynamic viscosity is defined using locally-equilibrium model of turbulence [1].

### Initial and boundary conditions

It is believed that at the beginning of time parameters of environment be considered to coincide with the values. To set up a low ground cover of grass-roots forest fire and massive speed blowing out of it. On the left edge of the wind speed is parallel to the surface of the earth and fire hearth bottom positioned at the start of the coordinate system. Then the process will be symmetrical about the coordinate plane  $Ox_1x_3$ , i.e.  $x_2=0$ . The primary assumptions and boundary conditions for the system of equations (1)-(8) are specified as follows:

$$t = 0: v_i = 0, T = T_e, c_\alpha = c_{ae}, T_s = T_e, \varphi_k = \varphi_{ke}, i = 1,2,3; k = 1,2,3; \alpha = \overline{1,5}. \quad (9)$$

$$x_1 = -x_{1e}: v_1 = V_e, v_2 = 0, \frac{\partial v_3}{\partial x_1} = 0, T = T_e, c_\alpha = c_{ae}, -\frac{c}{3k} \frac{\partial U_R}{\partial x_1} + cU_R/2 = 0; \quad (10)$$

$$x_1 = x_{1e}: \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0; \quad (11)$$

$$x_2 = x_{20}: \frac{\partial v_1}{\partial x_2} = 0, v_2 = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{\partial U_R}{\partial x_2} = 0; \quad (12)$$

$$x_2 = x_{2e}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0; \quad (13)$$

$$x_3 = 0: v_1 = 0, v_2 = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0, v_3 = v_{30}, T = T_g, |x_1| \leq \Delta, |x_2| \leq \Delta, \quad (14)$$

$$v_3 = 0, T = T_e, |x_1| > \Delta, |x_2| > \Delta;$$

$$x_3 = x_{3e}: \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0. \quad (15)$$

### Numerical method and results

System of equations (1)-(8) with the appropriate initial and boundary conditions, above, for numerical integration is reduced to the discrete form using control volume method [6]. Mesh equations arising in the process of discretization, resolved by using the *SIP* methods [7].

Solution algorithm given tasks includes splitting for physical processes that is initially calculated hydrodynamic picture, and then solved equations of chemical kinetics and chemical sources for scalar functions. When this step at a time to integrate the system of ordinary equations is chosen automatically. Calculation of pressure and velocity fields was iterative manner through SIMPLE algorithm [6].

As a result of numerical integration fields of velocity, component concentrations of the gaseous phase, temperature, volumetric fraction of solid phase are received at different points in time. The results showed that over time under the influence of surface forest fire rising temperature gas on the bottom edge of the forest canopy (fig. 2), is warming up FCM, evaporation, reduction of volume concentration of solid FCM, increase the amount of volatile combustible products of pyrolysis, mass concentration, decreased oxygen (fig. 3) and reducing the volume of phases (fig. 4) near the surface forest fire. Under the influence of high temperature in the surrounding area is warm canopy forest, dry moisture evaporation and decomposition of FCM. Figure 2-4 before changing temperature gas, mass concentrations of the gaseous phase and volumetric fractions over time. As a result, in the crown are volatile flammable forest products of pyrolysis. Throughout the process temperature of the gas phase above the solid phase. This confirms that the mechanism of energy transfer convectational prevailed over radiation.

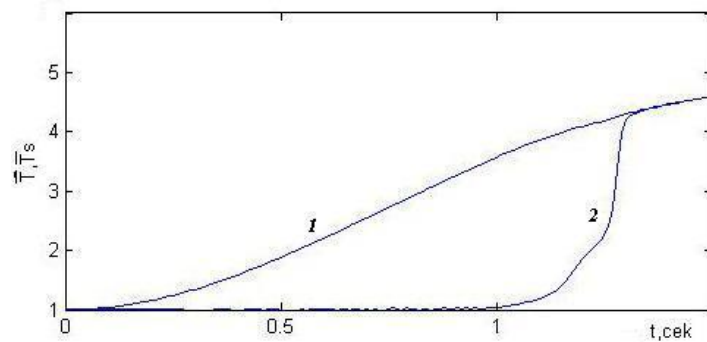


Figure 2. Temperature of gas phase (1 -  $\bar{T}$ ) and solid phase (2 -  $\bar{T}_s$ );  $\bar{T} = T/T_e$ ,  $\bar{T}_s = T_s/T_e$ .

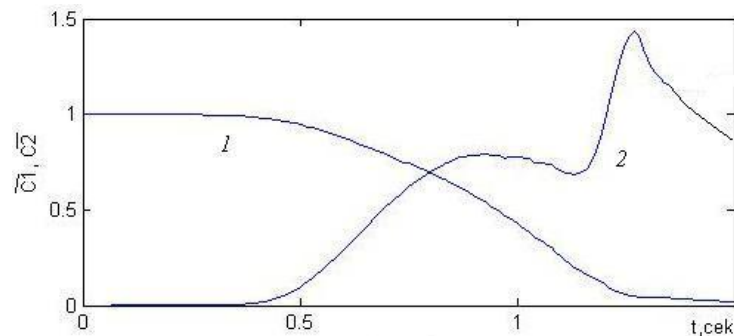


Figure3. Concentrations of oxygen (1-  $\bar{c}_1$ ) and combustible products of pyrolysis (2-  $\bar{c}_2$ );

$$\bar{c}_\alpha = c_\alpha / c_{1e}, c_{1e} = 0.23.$$

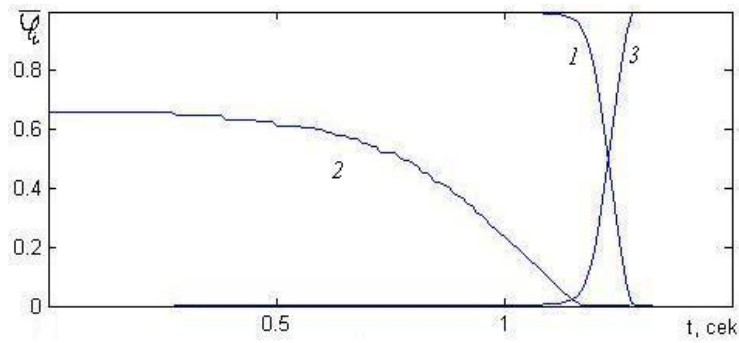


Figure.4 Volume fractions of condensed phases:

$$1 - \bar{\varphi}_1 = \varphi_1 / \varphi_{1e}, 2 - \bar{\varphi}_2 = \rho_2 \varphi_2 / \rho_c, 3 - \bar{\varphi}_3 = \rho_3 \varphi_3 / \alpha_c \rho_1 \varphi_{1e}.$$

Figures 5-7 show the temperature distribution, concentrations of oxygen and volatile product of pyrolysis of inception of pyrolysis ( $t = 1.0$ ) to the flammability ( $t = 1.5$  sec.).

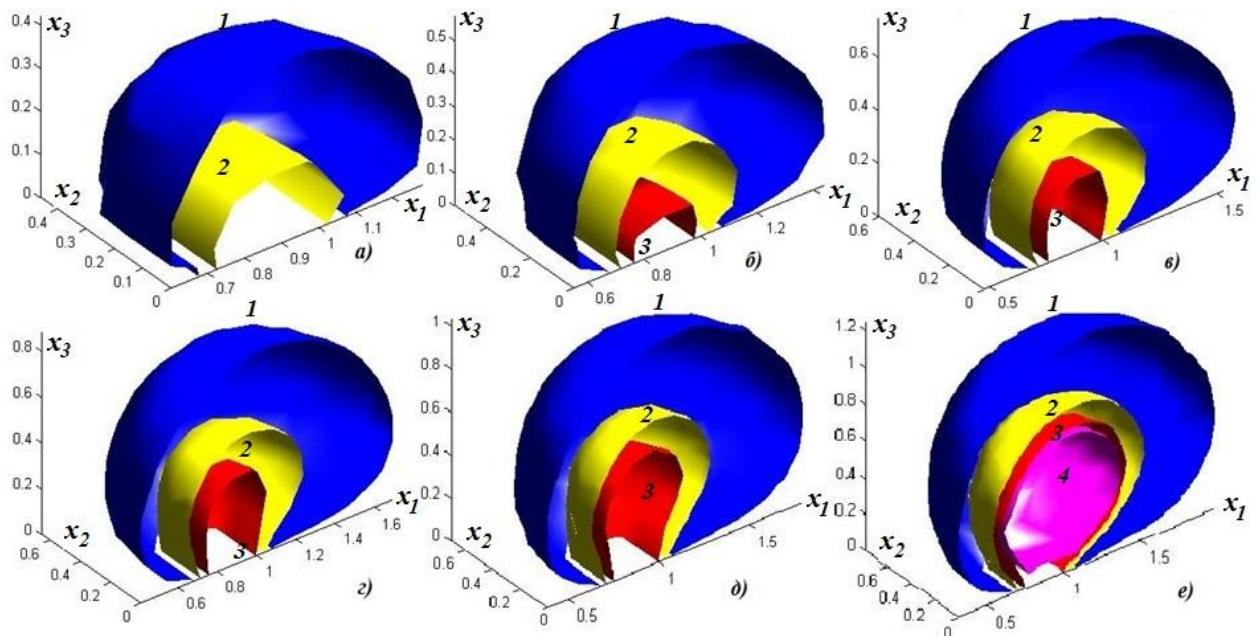


Figure.5. Isosurfaces of temperature  $\bar{T}$  (1 - 1.1, 2 - 2., 3 - 3., 4 - 4).

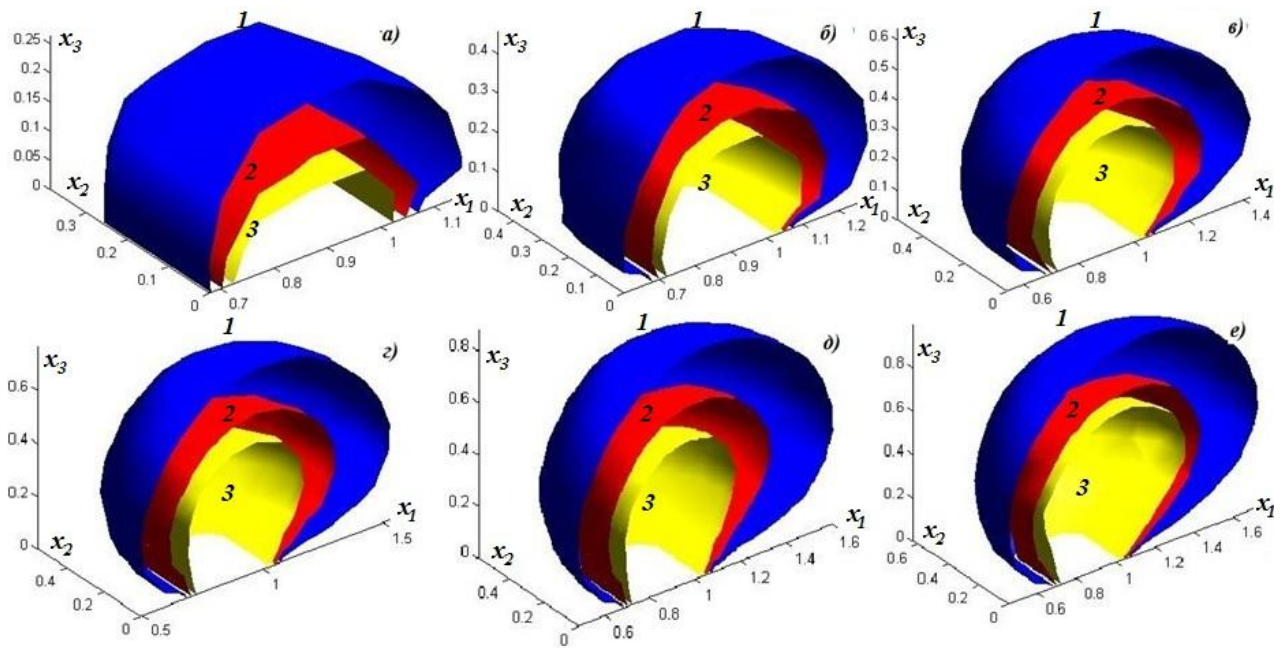


Figure.6. Isosurfaces of oxygen  $\bar{c}_1$  (1 - 0.9, 2 - 0.7, 3 - 0.5).

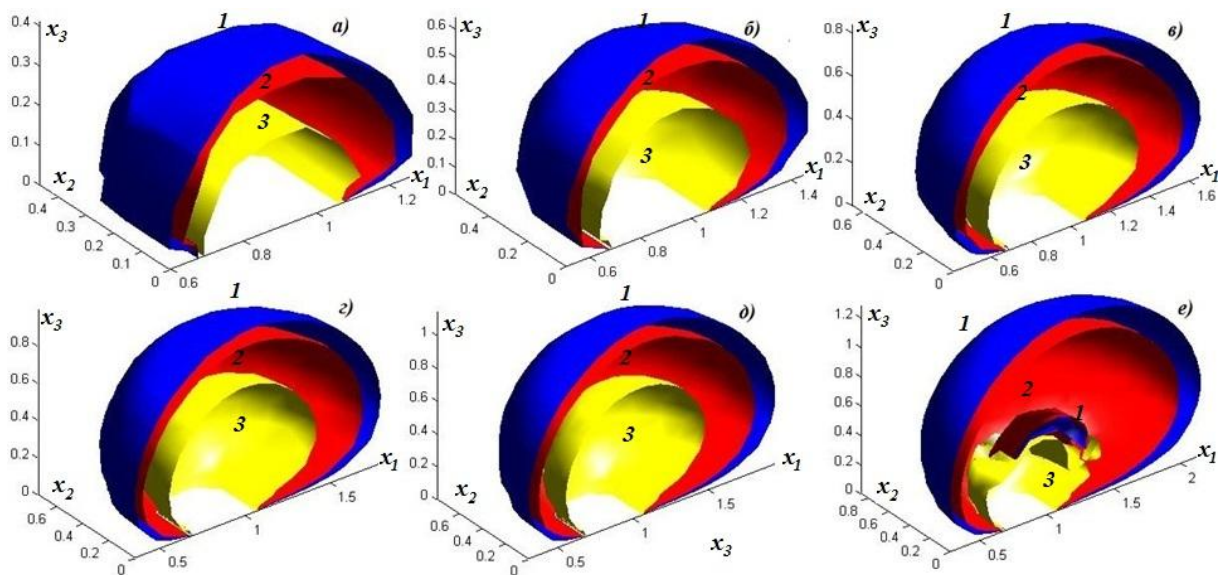


Figure7. Isosurfaces of volatile products of pyrolysis (1- 0.05, 2 - 0.1, 3 - 0.5).

The gaseous products of pyrolysis, released as a result of decomposition of FCM, ignited the space between the forest canopy cover and surface layer. They are burned down (reduced concentrations) at the bottom edge of the forest canopy. This also decreases the concentration of oxygen. From the moment of the ignition temperature of the gas and condensed phases are the same.

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