# FLAME HEIGHT MODEL OF A SPREADING SURFACE FIRE

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## Abstract
In this work, a simplified fire-spread model is presented. The mean features of a spreading surface fire-front such as rate of spread, tilt angle, flame height, are expounded. The model improves a previous work. The flame sub model was based on the McCaffrey law. In order to eliminate this weakness, it evolves thanks to simplified hypotheses and energy balance into an analytical relationship.
The model is tested on a set of experiments carried out at the Instituto Superior Técnico in 2003 at laboratory scale. The comparison between the model simulations and the experiments are in good agreement. The simplified physical model represents the behavior of the rate of spread and the flame height as well. Indeed, the model represents the increase of rate of spread when slope and/or wind speed increase. And, the flame height increase to a maximum and its decrease are explained by a relation between flame height and flame tilt angle, which is not provided by the literature.

## Introduction
Fires are the major source of forest destruction in the Mediterranean basin. Fire risk evaluation is crucial in regions such as the Mediterranean, where a sharp increase of fire events in forests has been observed during these last years [1].

Fire spread models and wildland fire calculation systems have been carried out in many scientific studies from the last decades. These previous works show that accurate estimation of parameters such as fire intensity, rate of spread and flame characteristics are important as they determine how a wildfire may be controlled, and they allow the evaluation of limits beyond which wildfire control becomes difficult. So, this knowledge in wildland fires is a tough but important problem. For instance, knowledge of flame height, flame width and flame angle together is essential in the estimation of the fire's thermal impact [2].

Following the classification of [3], three kinds of modelling, in accordance with the methods used in their construction, can be defined. The simplest models are the empirical ones, which do not to involve physical mechanisms [4]. Semi-physical models [5] are based upon the conservation of energy, but they do not distinguish the mode of heat transfer. Finally, physical models differentiate the various kinds of heat transfer in order to predict fire behaviour [6-10].

The physical fire spread model adopted in this paper is based on previous works [11, 12]. The aim of this article is to provide an expression of flame height ($H$) with the various characteristics of the fuel beds strata: surface mass of fuel, moisture content, high calorific value, surface to volume ratios…

The performance of this relationship is evaluated with the data obtained in a series of experiments under different wind and slope conditions [13].

**Simplified physical model**

Balbi et al. [11, 12] have presented a simplified physical model which provides most of the characteristic quantities of a fire front: rate of spread, tilt angle, temperature, depth, height. It derives from both assumptions and simplified physical balances. This simplified model gives good agreement with some surface and field scale fires. With a computational time which is faster than real time, this model intends to go into operational action mode. It is constituted by two uncoupled algebraic equations which provide respectively the flame tilt angle and the rate of spread,

$$\tan\gamma = \tan\alpha + \frac{U}{u_0} \tag{1}$$

$$R = R_0 + A\,R\,\frac{1+\sin\gamma-\cos\gamma}{1+\frac{R\cos\gamma}{r_0}} \tag{2}$$

with four parameters ($R_0$, $A$, $u_0$, $r_0$), and

$$R_0 = \frac{e}{\sigma}\frac{R_{00}}{1+am} \tag{3}$$

$$A = \inf\left(\frac{S}{4};1\right)A_0\ ;\ S = s\,e\,\beta$$

$$A_0 = \frac{\chi_0\,\Delta H}{4c_p\Delta T(1+am)} \tag{4}$$

$$u_0 = u_{00}\,\frac{\sigma}{\tau} \tag{5}$$

$$r_0 = 12\,R_0 \tag{6}$$

$$T = T_a + \frac{\Delta H\,(1-\chi)}{(st+1)c_{pa}} \tag{7}$$

$$\chi = \frac{\chi_0}{1+\frac{R\cos\gamma}{r_0}} \tag{8}$$

In this work, only the case of the head fire ($\gamma > 0$) is considered, i.e. when the flame tilts to the unburnt vegetation. In the rear fire case, the relationship for $R$ is different.

Even though this model is simple and efficient, it can be improved. Equations (1) and (2) will not change because they are the core of the model. The improvements will concern relations (3) to (6).

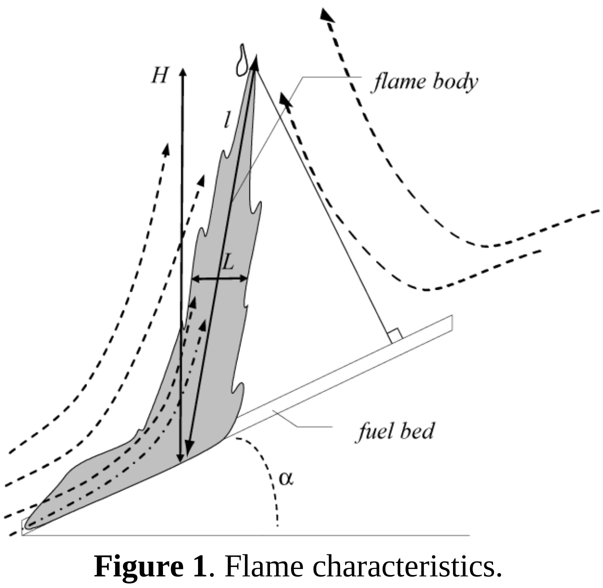


**Figure 1**. Flame characteristics.

**Parameters improvement**

*Parameter denoted $r_0$*

Equation 6 led us to eliminate the parameter $r_0$ in eq. (2), and the model gave satisfactory results. Nevertheless, there is no reason why the parameter $r_0$ should be in proportion to $R_0$. $r_0$ is derived from the law of the fraction radiation, which expresses the decrease with the flame surface-volume ratio

$$\chi = \frac{\chi_0}{1 + \mu \frac{V}{S}}$$

with

$$\frac{V}{S} = \frac{\frac{1}{2} H\, L \times 1}{l \times 1} = \frac{1}{2} \mathrm{L} \cos\gamma$$

At stationary state $L = R\tau$, where $L$ is the flame depth, $H$ the flame height from the ground, and

$$\chi = \frac{\chi_0}{1+\mu\,\tau\,R\cos\gamma} = \frac{\chi_0}{1+\frac{R\cos\gamma}{r_0}} \tag{8}$$

with $r_0 = \frac{1}{\mu\,\tau}$.

Following [14], one can consider that $\tau = \frac{\tau_0}{s}$ with $\tau_0$ = 75591 s.m$^{-1}$, so eq.(6) is replaced by the following one

$$r_0 = \mathrm{s}\, r_{00} \tag{9}$$

The parameter $r_{00}$ is a universal constant fitted on the set of experiments [15, 13], and is estimated at $2.5\times10^{-5}$.

*Parameter denoted $R_0$*

Equation 3 is unchanged. But instead of keeping $R_{00}$ as a constant to fit, the radiation of the base of the flame is expressed by

$$R_0 = \frac{e}{\sigma} \frac{BT^4}{c_p \Delta T + m\Delta h} \tag{10}$$

*Parameter denoted $u_0$*

In [11], $u_0$ is derived from mass balance in the flame

$$u_0 = \frac{2(st+1)}{\rho} \frac{\sigma}{\tau} \tag{11}$$

In fact fuel load $\sigma$ is the useful one, $\sigma_u$, i.e. the fuel proportion in combustion. It is more right to rewrite eq. (11) and replace it by

$$u_0 = \frac{2(st+1)}{\rho} \frac{\sigma_u}{\tau} \tag{12}$$

and $\frac{\sigma_u}{\sigma} = \frac{e_u}{e}$ where $e_u$ is the useful depth, i.e. which is in combustion. So, eq.(12) can be written as follow

$$u_0 = 8\frac{\rho_v}{\rho_a}\frac{st+1}{\tau_0}\frac{T}{T_a}\frac{S\,\sigma}{4\rho_v}\frac{e_u}{e}$$

then,

$$u_0 = u_{00}\frac{S}{4}\frac{e_u}{e} \quad (13)$$

with

$$u_{00} = 8\frac{st+1}{\tau_0}\frac{\rho_v}{\rho_a}\frac{T}{T_a} \quad (14)$$

$e_u$ represents the fuel bed thickness really preheated by the flame. The radiated energy by the flame is absorbed to a distance $\delta$, called optical depth, which is defined by [16] (see Fig. 2)

$$\delta = \frac{4}{\beta s} \quad (15)$$

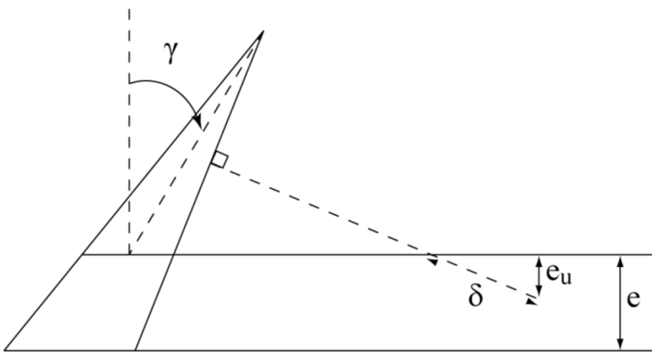

**Figure 2**. Optical depth.

Two cases can be distinguished:

Case #1: $e_u < e$

This occurs for $\delta < e$ which means for $\frac{S}{4} > 1$ or for $\frac{S}{4} < 1$ and $\gamma < \arcsin\left(\frac{S}{4}\right)$.

So, the fuel bed thickness preheated by the flame, $e_u$, can be expressed by $e_u = \delta \sin\gamma$, and $\frac{e_u}{e} = \frac{4}{S}\sin\gamma$. Then, eq.(14) is written again with $u_0 = u_{00}\sin\gamma$.

Case #2: $e_u = e$

This means that $\frac{S}{4} < 1$ and $\gamma > \arcsin\left(\frac{S}{4}\right)$. So, eq.(14) is replaced by $u_0 = u_{00}\frac{S}{4}$.

*Parameter denoted A*

Equation 4, $A_0$, represents the part of flame radiation. Now, it is interesting to represent the part of this heat flux that is absorbed by the fuel bed. One can distinguish the two previous cases:

Case #1: the heat flux is totally absorbed by the fuel

$$A = A_0$$

Case #2: the heat flux is absorbed in proportion $\frac{d}{\delta}$ (see Fig. 3)

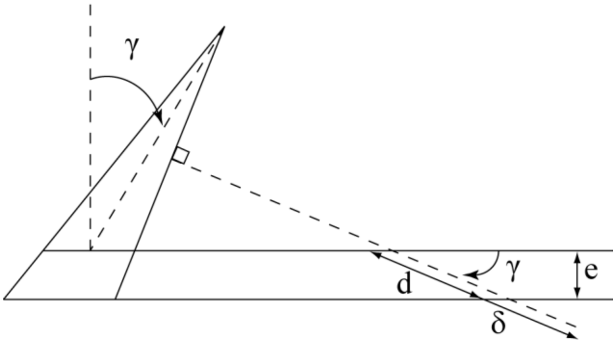


**Figure 3**. Absorbed heat flux fraction.

$$\frac{d}{\delta} = \frac{{}^{e}/_{\sin\gamma}}{{}^{4}/_{\beta s}} = \frac{{}^{S}/_{4}}{\sin\gamma}$$

and

$$A = \frac{{}^{S}/_{4}}{\sin\gamma} A_0$$

Equation (4) is derived with the hypothesis of the radiant panel with an infinite length. In order to free from this limitation, [17] have introduced a corrective term when the panel has a finite width, *W*. So, eq.(4) can be improved and replaced by

$$A_0 = \frac{\chi_0}{4\left(c_p\,\Delta T + m\Delta h\right)} Y$$

with $Y = \tanh\left[\frac{2}{3}\left(\frac{W}{l}\right)^{1/3}\right]$, where *l* is the length of the flame.

**Improved model**
The core of the model is unchanged

$$\tan\gamma = \tan\alpha + \frac{U}{u_0} \quad (1)$$

$$R = R_0 + A\,R\,\frac{1+\sin\gamma-\cos\gamma}{1+\frac{R\cos\gamma}{r_0}} \quad (2)$$

But, on the other hand, the laws of the parameters are changed

$$r_0 = \mathrm{s}\,r_{00}$$

$$R_0 = \frac{e}{\sigma}\frac{BT^4}{c_p\Delta T + m\Delta h}$$

$$T = T_a + \frac{\Delta H\,(1-\chi)}{(st+1)c_{pa}}$$

$$u_{00} = 8\,\frac{st+1}{\tau_0}\frac{\rho_v}{\rho_a}\frac{T}{T_a}$$

$$A_0 = \frac{\chi_0\,\Delta H}{4\left(c_p\Delta T + m\Delta h\right)}\mathrm{Y}$$

$$Y = \tanh\left[\frac{2}{3}\left(\frac{W}{l}\right)^{1/3}\right]$$

Case #1

$$\frac{S}{4} > 1 \text{ or for } \frac{S}{4} < 1 \text{ and } \gamma < \arcsin\left(\frac{S}{4}\right)$$
$$u_0 = u_{00}\sin\gamma$$
$$A = A_0$$

Case #2

$$\frac{S}{4} < 1 \text{ and } \gamma > \arcsin\left(\frac{S}{4}\right)$$
$$u_0 = u_{00}\,\frac{S}{4}$$
$$A = \frac{S/4}{\sin\gamma}A_0$$

**Two fire-spread regimes**

The literature shows the appearance of two different regimes of fire dynamics [18]:

- a fast regime where $\frac{R}{R_0}$ becomes very high when the tilt angle increases;
- a slow regime where $\frac{R}{R_0}$ is bounding when the tilt angle increases.

Equation 2 provides then two approximated relations for the asymptotic regime when $\gamma$ tends towards $\frac{\pi}{2}$.

*The fast regime: $R_0 \ll R$*

In this case, eq.(2) can be expressed by

$$1 \approx A\,\frac{1+\sin\gamma-\cos\gamma}{1+\dfrac{R\cos\gamma}{r_0}}$$

and

$$1 + \frac{R\cos\gamma}{r_0} \approx A(1+\sin\gamma-\cos\gamma) \xrightarrow[\gamma\to\frac{\pi}{2}]{} \frac{R\cos\gamma}{r_0} \approx 2A-1$$

This expression is verified for $A > ½$, and eq.(2) can be approximated by

$$R \approx r_0(2A-1)\tan\gamma$$

*The slow regime: $R \ll r_0$*
Here, eq.(2) can be expressed by

$$R \approx R_0 + AR(1+\sin\gamma-\cos\gamma)$$

then,

$$R \approx \frac{R_0}{1-2A}$$

This is verified for $A < ½$.
Figure 4 shows the two different fire regimes. One can notice that in the slow regime ($A < ½$) radiation becomes weak and convection is not negligible. Moreover in that case, if convection is negligible and the flame tilt flame is weak, no propagation occurs. It is the reason why only the fast regime ($A > ½$) should be considered in this paper.

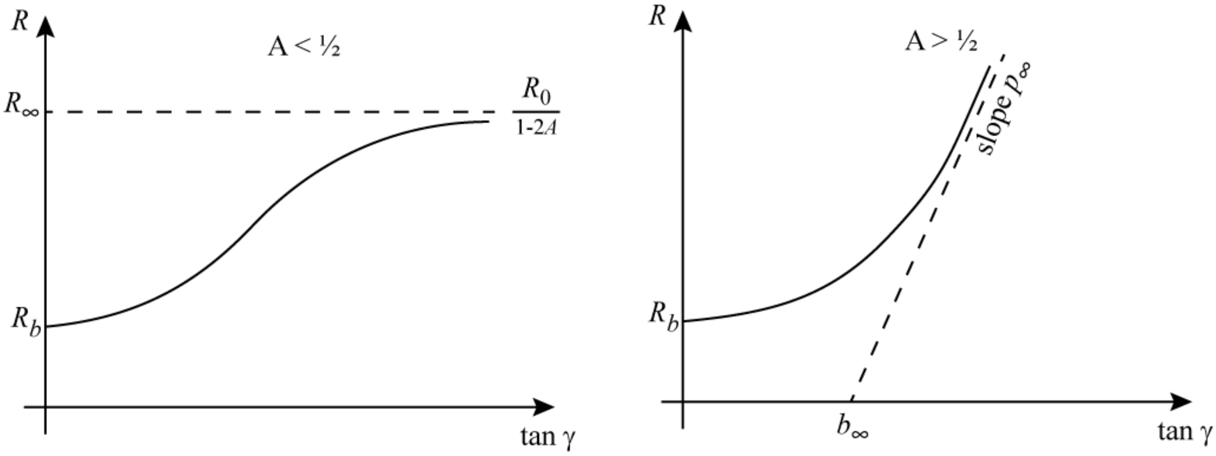


**Figure 4.** Two different regimes for the rate of spread.

**Flame height sub-model**
As shown on Fig. 1, the flame height is defined as the height of the equivalent radiant panel. The radiative flux of a unit surface is given by $\varepsilon BT^4$, where $\varepsilon$ is the equivalent flame emissivity and $T$ the mean flame temperature given by eq.(7). So, the linear radiative flux is $\varepsilon BT^4 \times l \times 1$, which is, also, expressed by $\frac{1}{2}\chi Q$.

$$\varepsilon BT^4 l = \frac{1}{2}\chi Q \qquad (16)$$

with $l = \frac{H}{\cos\gamma}$, $\chi$ given by eq. 8, and

$$Q = \Delta H\ L\dot{\sigma} = \Delta H\ R\tau\frac{\sigma_u}{\tau} = \Delta H\ R\sigma_u = \Delta H\ R\sigma\frac{e_u}{e} = \Delta H\ R\sigma\frac{4}{S}\frac{u_0}{u_{00}} \qquad (17)$$

At this point, only emissivity has to be expressed. When flame depth tends towards 0, $\varepsilon$ tends towards 0. And, when flame depth tends towards infinity, $\varepsilon$ tends towards 1.
So, a law which conveys this trend and is linked to radiative factor $\chi$:

$$\varepsilon = a\frac{\frac{R\cos\gamma}{r_0}}{1+\frac{R\cos\gamma}{r_0}} \tag{18}$$

When $\gamma$ tends towards $\frac{\pi}{2}$ and if $A > \frac{1}{2}$, $\frac{R\cos\gamma}{r_0}$ tends towards $2A-1$. So, $\varepsilon$ tends towards $a\frac{2A-1}{2A} \approx 1$, and $a = \frac{1}{1-\frac{1}{2A}}$. Eq. (20) can be rewritten as

$$\varepsilon = \left(\frac{1}{1-\frac{1}{2A}}\right)\frac{\frac{R\cos\gamma}{r_0}}{1+\frac{R\cos\gamma}{r_0}} \tag{18'}$$

Then eq.(17) is written as follows

$$\left(\frac{1}{1-\frac{1}{2A}}\right)\frac{\frac{R\cos\gamma}{r_0}}{1+\frac{R\cos\gamma}{r_0}}BT^4\frac{H}{\cos\gamma} = \frac{1}{2}\frac{\chi_0}{1+\frac{R\cos\gamma}{r_0}}\Delta H\, R\sigma\frac{4}{S}\frac{u_0}{u_{00}}$$

$$H = \frac{1}{2}\chi_0\Delta H\left(1-\frac{1}{2A}\right)\frac{r_{00}}{BT^4}s\beta e\rho_v\frac{4}{S}\frac{u_0}{u_{00}}$$

So, flame height can be expressed as

$$H = \frac{H_0}{BT^4}\frac{u_0}{u_{00}} \tag{19}$$

with

$$H_0 = 2\chi_0\Delta H\left(1-\frac{1}{2A}\right)r_{00}\rho_v \tag{20}$$

The two distinguished cases are expressed as
Case #1

$$H = \frac{H_0}{BT^4}\sin\gamma$$

$$H_0 = 2\left(\chi_0\Delta H - \frac{2}{Y}\left(c_p\Delta T + m\Delta h\right)\right)r_{00}\rho_v$$

Case #2

$$H = \frac{H_0}{BT^4}\frac{S}{4}$$

$$H_0 = 2\left(\chi_0\Delta H - \frac{2}{Y}\frac{S/_4}{\sin\gamma}\left(c_p\Delta T + m\Delta h\right)\right)r_{00}\rho_v$$

## Results and discussion

The set of experimental data [13] concerns experiments carried out at the Instituto Superior Tecnico (I.S.T.) of Lisboa (Portugal) under both combined wind and upslope conditions. The wind speed values covered a range between -3 to 3 m/s and the movable tray could be set at angles from -15 up to 15° with upslope orientation. The fuel bed made up of needles of *Pinus pinaster* is 0.70 *m* wide and 2 m long with a load of 0.5 kg/m$^2$ on a dry basis. Two fuel moisture contents (FMC) of 10% (± 1%) and 18% (± 1%) were studied. Only the configurations for a heading fire are used in this paper, i.e. positive wind speed and slope ranged from 0 to 15° by step of 5°. For all the following figures, the line represents the predicted results and the symbols represent the experimental data.

Figure 5 and 6 show the rate of spread (ROS) versus the tilt angle $\gamma$. A substantial spread rate increase can be seen with increasing slope and wind. This increase in the rate of spread, due to a tendency of the flame to be deflected towards the slope is observed. It can be seen that these results are in agreement with the experimental data. On Fig. 5, one point ($U$ = 0.5 m/s; $\alpha$ = 0°) is alone and three subset of data are clearly observed. For wind speed of 2 and 3 m/s, two measures ($U$ = 2 m/s; $\alpha$ = 15°) and ($U$ = 3 m/s; $\alpha$ = 5°) are not in accordance with the other measures. The model represents the increase of rate of spread when slope and/or wind speed increase.

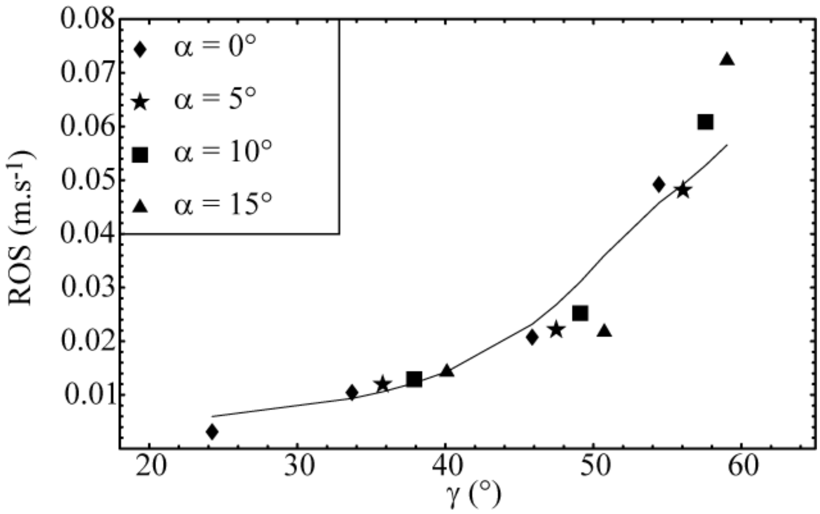


**Figure 5**. Rate of spread for a FMC of 10%.

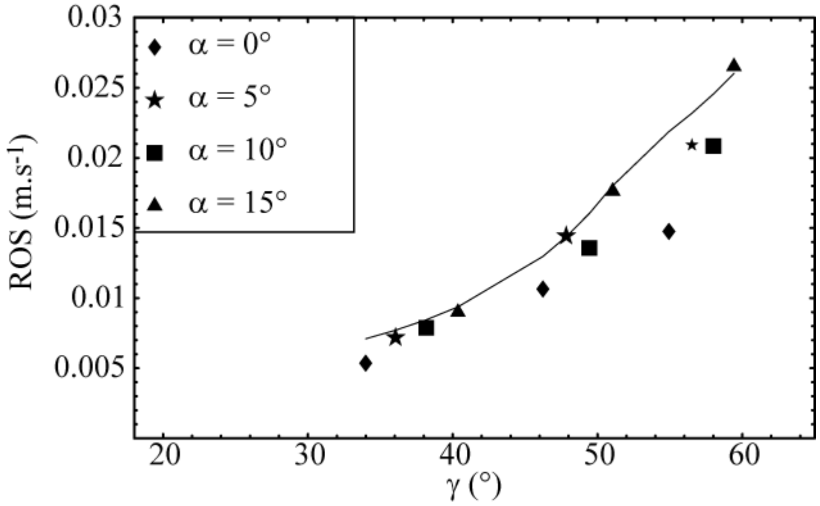


**Figure 6**. Rate of spread for a FMC of 18%.

On Fig. 6, the fuel moisture content is changed and the computation provides results which are slightly over-estimated. The focus can be done on the fact that the model provides good agreement with experimental data when only the FMC is changed, i.e. no physical and model parameters are modified. FMC has an influence on rate of spread. Indeed rate of spread decreases when FMC increases.

[13] observes that flame height steeply increases initially, reaches a maximum of about 45 cm for wind speed of around 1.5 m.s$^{-1}$, and then decreases (Figs. 7 and 8). Fuel moisture content has a small but clear influence on flame height because combustion intensity decreases with fuel moisture content.
The analytical relationship (19)-(20) allows to observe that "bell-shaped" behavior. Indeed, the model is in agreement with experimental data. The deviation observed on Fig. 7 and Fig.8 between predicted and observed flame height is essentially due to the difficulty to obtain accurated measures. Indeed, the standard deviation for flame height ranges from 8 to 15 cm for a FMC of 10% and from 7 to 13 cm for a FMC of 18%.

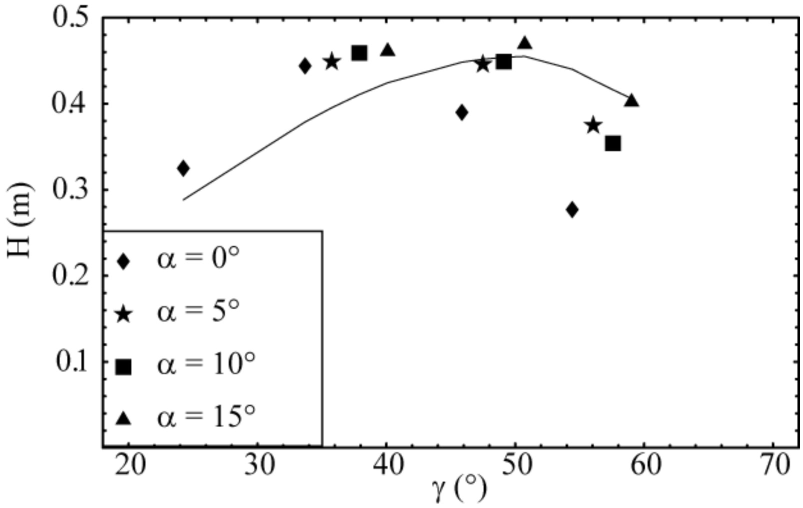


**Figure 7**. Flame heights for a FMC of 10%.

The model represents the observed diminution of the flame height too when fuel moisture content increases.

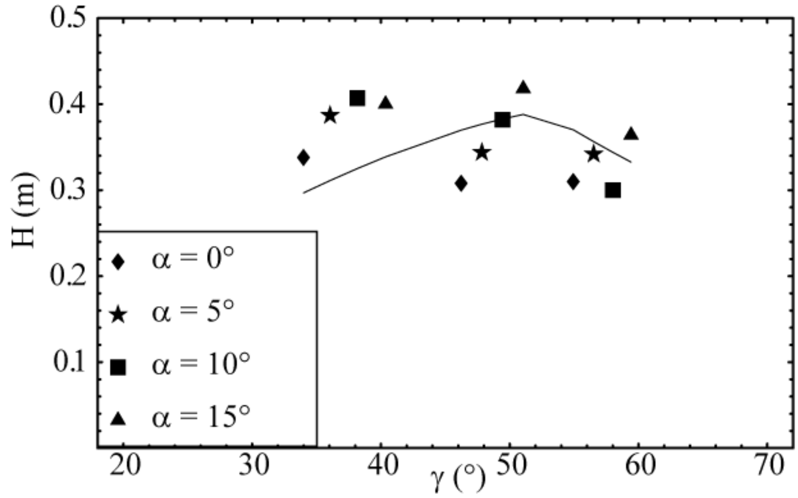


**Figure 8**. Flame heights for a FMC of 18%.

## Conclusion

This work is a new stage in the aim of Corte's Forest Fire Research Team which consists in obtaining a fire spread model which provides rate of spread, flame geometrical characteristics and thermodynamic quantities.

The model obtained, with radiation as the most important mechanism of the fire spread, is simple, quasi-physical without any parameter fitting.

It can be noticed that a relationship of flame height, thus flame length, with some fuel characteristics and with the environment (slope, wind and humidity) is provided. This relationship allows to explain this "bell-shaped" behavior observed between flame height and flame tilt angle, which is not provided by literature.

In a future work, convection heating due to the hot gases released by combustion will be taken into account for wind-driven fires.


## Acknowledgements

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## Nomenclature

| *Symbol* | *Units* | *Description* |
|---|---|---|
| $A$ | | Ratio between incident radiant energy and ignition energy of wet fuel |
| $B$ | $W.m^{-2}.K^{-4}$ | Stefan-Boltzmann constant |
| $C_p$, $C_{pa}$ | $J.kg^{-1}.K^{-1}$ | Specific heat of the vegetative fuel (1900), of the air (1150) |
| $e$ | m | Fuel bed thickness (0.04 m) |
| $H$, $l$, $L$ | m | Flame height, Flame length, Flame base depth |
| $m$ | % | Dead fine fuel moisture content (10%, 18%) |
| $R$ | $m.s^{-1}$ | Rate of spread |
| $R_0$ | $m.s^{-1}$ | Part of the rate of spread due to radiation of the burning fuel bed |
| $r_0$ | $m.s^{-1}$ | ROS factor |
| $s$ | $m^{-1}$ | Surface area to volume ratio of fuel elements (400 $m^{-1}$) |
| $S$, $s_t$ | | Leaf area index, stoichiometric coefficient (9) |
| $T$, $T_a$, $T_i$ | K | Flame temperature, ambient air temperature, ignition temperature |
| $U$ | $m.s^{-1}$ | Normal wind velocity |
| $u$, $u_0$ | $m.s^{-1}$ | Upward gas velocity with slope, on a flat terrain |
| $W$ | m | Fire front width |
| $\alpha$, $\gamma$ | ° | Slope angle, flame tilt angle |
| $\beta$ | | Packing ratio |
| $\chi$, $\chi_0$ | | Radiative heat loss fraction, Radiant factor (0.32) |
| $\Delta h$ | $J.kg^{-1}$ | Heat of latent evaporation |
| $\Delta H$ | $J.kg^{-1}$ | Heat fuel combustion (1.4 $10^7$ $J.kg^{-1}$) |
| $\Delta T$ | K | $T_i - T_a$ = 300 K |
| $\rho_v$, $\rho_a$ | $kg.m^{-3}$ | Fuel density (630 $kg.m^{-3}$), air density (1 $kg.m^{-3}$) |
| $\sigma$ | $kg.m^{-2}$ | Fuel load (0.5 $kg.m^{-2}$) |
| $\tau$ | s | Flame residence time |
| $\tau_0$ | $s.m^{-1}$ | Anderson's flame residence time coefficient (75591 $s.m^{-1}$) |

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